**A calculation of the commutator of spin angular momentum and angle**

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A classical definition of the angular momentum is written as

 (1)

where is a radial vector, and being the momentum. That is,

 (2)

 (3)

By these definitions, we can form a quantum mechanical definition of angular momentum.

 (4)

where are unit vectors each being along the xyz axes.

Now, we will concentrate on the z-component.

 (5)

Then what will the corresponding variable that will constitute the Heisenberg’s uncertainty principle be. If we write the unit system for the angular momentum like

 (6)

Then the corresponding variable cannot be any variable having meaningful dimension, but the dimensionless variable. The most probable variable is the angle.

 (7)

Now, let us proceed to the calculation of the commutational relationship of the and.

The commutator between them can be calculated by the following way where the function means an arbitrary function.

 (8)

What is needed for us to proceed is the calculation of the first term.

 (9)

The second term in the equation (9) cancels out the second term in equation (8), so that what is remaining is the task of calculation the first term in the equation (9).

By this elementary calculus, we can obtain the following.

 (9)

Therefore, the commutator becomes

 (10)

As a result, the uncertainty principle about the angular momentum, with a little modifying constant, becomes



Therefore, if we do not know the angle variable completely, then the lowest angular momentum cannot help being . This is the theoretical reason fro the origin of electron spin.

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