**Calculations of various polynomials of Hamiltonian operator**

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The time-dependent Schroedinger equation is

 (1)

If we multiply the both sides by, then we obtain

 (2)

Therefore, the formal solution for can be written as

 (3)

Stepping ahead the above equation by time sliceyields

 (4)

Since the wave function is a complex function and the exponential operator may be written by a sum of cosine and sine function, we have the following set of equations.

 (5)

It is clear that

 (6)

And we know the following formula.

 (7)

The equation (4) can now be rewritten as

 (8)

A calculation of the right side hand shows that the real part equation becomes as

 (9)

The imaginary part becomes

 (10)

By using matrix formalism, we can combine the equations (9) and (10).

 (11)

The matrix in the right side hand of the above equation is completely unitary because

 (12)

The cosine function and sine function can be expanded by infinite power series.

 (13)

 (14)

By truncating at various power of the equation (13) and (14), we can derive various levels of approximation of equation (11).

 **First order approximation**

In this level, we approximate the infinite polynomial by using until first order term.

 (15)

The transformation equation (11) may become

 (16)

Since the determinant of the transformation matrix is

 (17)

the order of deformation error is proportional to the second power of time slice .

 **Second order approximation**

 (18)

The transformation equation (11) may become

 (19)

Since the determinant of the transformation matrix is

 (20)

the order of deformation error is proportional to the fourth power of time slice .

 **Third order approximation**

 (21)

The transformation equation (11) may become

 (22)

Since the determinant of the transformation matrix is



(23)

the order of deformation error is proportional to the fourth and sixth powers of time slice ,.

 **Fourth order approximation**

 (24)

The transformation equation (11) may become

 (25)

Since the determinant of the transformation matrix is



(26)

the order of deformation error is proportional to the sixth and eighth powers of time slice ,

 **Fifth order approximation**

 (27)

The transformation equation (11) may become



(28)

Since the determinant of the transformation matrix is



(29)

the order of deformation error is proportional to the sixth, eighth and tenth powers of time slice ,and

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