Quantum Hall effect in intersubband transitions of GaN/AlGaN Quantum well without Landau levels

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CHAPTER 21

Quantum Hall effect in intersubband transitions of GaN/AlGaN Quantum well without Landau levels

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Title: Quantum Hall effect in intersubband transitions of GaN/AlGaN Quantum Well without Landau levels
Liubov E. Lokot

Abstract In the article the Quantum Spin Hall effects are shown to be related with intraband transitions of bulk GaN. In the framework of the effective mass theories we have solved the Schrödinger equation if the topological insulator transformation is achieved. In the article the Schrödinger equation are shown to be connected with the symmetry point group of $C_{6v}$. The exact solutions of the Schrödinger equations as well as Quantum Spin Hall effect of intersubband transitions of bulk GaN are found. For the hexagonal symmetry of GaN the Effective Hamiltonian based on $C_{6v}$ point symmetry group was found. Derivation of expressions of momentum matrix elements of the both intersubband phototransitions as well as interband phototransitions for bulk GaN are considered. In the article for Quantum Hall effect of intersubband phototransitions of bulk GaN the expressions of Berry curvature as well as Hall conductivities have been found when the topological insulator transformation is achieved.

URL: https://figshare.com/authors/Liubov_Lokot/5212580
https://www.zotero.org/liubov_e_lokot/items/
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1 Derivation of effective Hamiltonian of wurtzite band dispersion
The strong strain induces warping considerable Rashba spin-orbit (SO) interactions (SOI) on fermi surfaces that may transform the valence band system into a topological insulator (TI). The latter testify about the significant increased value of Quantum Spin Hall (QSH) effects. The basic idea is as follows. A set of hole bands that in absence SO coupling belongs to orbital momentum $L$, in presence of SO coupling have been described by the total angular momentum $J = L + S$. The TI state can be achieved in semiconductors with inverted bands and in usually driven by the intrinsic SOI arising from heavy host atoms [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

An irreducible representation for the orbital angular momentum $j$ can be built from the formulas

\[ \chi_j(\varphi) = \sum_{m} D_{j,mm}(\varphi) = \sum_{m=-j}^{j} \exp i m \varphi = \frac{\sin \left( j + \frac{1}{2} \right) \varphi}{\sin \frac{\varphi}{2}} = 1 + 2 \cos \varphi, \]

\[ \chi(C_6) = \chi(\frac{2\pi}{6}) = 2, \]
\[ \chi(C_3) = \chi(\frac{2\pi}{3}) = 0, \]
\[ \chi(C_2) = \chi(\frac{2\pi}{2}) = -1, \]
Table 1. The irreducible representation of $C_{6v}$ [15, 16, 17].

<table>
<thead>
<tr>
<th>$C_{6v}$</th>
<th>$e$</th>
<th>$C_2$</th>
<th>$2C_3$</th>
<th>$2C_6$</th>
<th>$3\sigma$</th>
<th>$3\sigma'$</th>
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<tbody>
<tr>
<td>$\tau_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>z</td>
</tr>
<tr>
<td>$\tau_2$</td>
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<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
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<td>1</td>
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<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>$\tau_5$</td>
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<td>-1</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\tau_6$</td>
<td>2</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>$x, y$</td>
</tr>
<tr>
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<td>1</td>
<td>$\sqrt{3}$</td>
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<td>0</td>
</tr>
<tr>
<td>$\tau_8$</td>
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<td>0</td>
<td>-1</td>
<td>$-\sqrt{3}$</td>
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<td>0</td>
</tr>
<tr>
<td>$\tau_9$</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$\tau_7 + \tau_9$</td>
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<td>0</td>
<td>-1</td>
<td>$\sqrt{3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\chi_v$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. The irreducible representation of $C_{6v}$ [15, 16, 17].

<table>
<thead>
<tr>
<th>$C_{6v}$</th>
<th>$\Gamma_1 + \Gamma_5$</th>
<th>$E$</th>
<th>$C_2$</th>
<th>$2C_3$</th>
<th>$2C_6$</th>
<th>$3\sigma_v$</th>
<th>$3\sigma'_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_v^x(g)$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$g^x$</td>
<td>$E$</td>
<td>$E$</td>
<td>$C_3$</td>
<td>$C_3$</td>
<td>$E$</td>
<td>$E$</td>
<td></td>
</tr>
<tr>
<td>$\chi_v(g^x)$</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}[+]$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>$2\Gamma_1 + \Gamma_5 + \Gamma_6$</td>
</tr>
<tr>
<td>$\frac{1}{2}[-]$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>$\Gamma_2 + \Gamma_5$</td>
</tr>
</tbody>
</table>

Table 3. The vector irreducible representation of $C_{6v}$ [15, 16, 17].

<table>
<thead>
<tr>
<th>$C_{6v}$</th>
<th>$E$</th>
<th>$C_2$</th>
<th>$2C_3$</th>
<th>$2C_6$</th>
<th>$3\sigma_v$</th>
<th>$3\sigma'_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_v$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{1}{2}[+]$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\frac{1}{2}[-]$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

$$
\begin{align*}
\sigma &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \\
\sigma' &= \begin{bmatrix} x' \\ -y' \\ z' \end{bmatrix}, \\
\sigma' &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\end{align*}
$$

The direct production of two irreducible representations of both wave vector and wave function of difference $\kappa - \Gamma$ expansion with taken into account time inversion can be expanded on

$$
p^a: (\Gamma_1 + \Gamma_5) \times (\Gamma_2 + \Gamma_3) = \Gamma_5 \times \Gamma_5, \\
p^a p^\beta + p^\beta p^a: (2\Gamma_1 + \Gamma_5 + \Gamma_6) \times (2\Gamma_1 + \Gamma_5 + \Gamma_6) = 4\Gamma_1 \times \Gamma_1 + \Gamma_5 \times \Gamma_5 + \Gamma_6 \times \Gamma_6, \\
p^a p^\beta = i \hbar c \left( \frac{\partial A^\alpha}{\partial x^\gamma} - \frac{\partial A^\gamma}{\partial x^\alpha} \right) = -i \hbar c \sum_{\beta} e_{\alpha \beta \gamma} H_{\gamma}, \\
\sigma_{\gamma, H_{\gamma}}: (\Gamma_2 + \Gamma_3) \times (\Gamma_2 + \Gamma_5) = \Gamma_2 \times \Gamma_2 + \Gamma_5 \times \Gamma_5.
$$
The bases functions and operators of irreducible representation of $\mathcal{C}_{0y}$ [15, 16, 17].

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$k_z^2, k_x^2, e_{zz}, e_i; \sigma_x k_x + \sigma_y k_y$</th>
<th>$J_z^2, I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>$k_z^2, k_x^2, e_{zz}, e_i; \sigma_x k_x + \sigma_y k_y$</td>
<td>$J_z^2, I$</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>$\sigma_x k_x^2, \sigma_y k_y^2$</td>
<td>$J_z^2, I$</td>
</tr>
<tr>
<td>$\Gamma_3$</td>
<td>$\sigma_x k_x$, $\sigma_y k_y$</td>
<td>$J_z$</td>
</tr>
<tr>
<td>$\Gamma_4$</td>
<td>$\sigma_x k_x, \sigma_y k_y$</td>
<td>$J_z$</td>
</tr>
<tr>
<td>$\Gamma_5$</td>
<td>$\frac{k_z^2, k_x^2, e_{zz}, e_i; \sigma_x k_x + \sigma_y k_y}{J_z^2, I}$</td>
<td>$J_z^2, J_z^2$</td>
</tr>
<tr>
<td>$\Gamma_6$</td>
<td>$k_z^2, k_x^2, e_{zz}, e_i; \sigma_x k_x + \sigma_y k_y$</td>
<td>$J_z^2, J_z^2$</td>
</tr>
</tbody>
</table>

\[
k_{\pm} = k_x \pm i k_y, J_\pm = \frac{1}{\sqrt{2(J_{\pm} \pm J_y)}}, 2[J_{\pm} J_y] = J_{\pm} J_y + J_y J_{\pm},
\[
\sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i \sigma_y), e_{\pm z} = e_{xx} \pm i e_{yy}, e_{\pm} = e_{xx} \pm 2i e_{xy} + e_{yy},
\]

\[
k_{\pm} = k_x \pm i k_y, J_{\pm} = \frac{1}{2}(k_x \pm i k_y), k_0^2 = k_x^2 + k_y^2,
\]

\[
e_{\pm z} = e_{xx} \pm i e_{yy}, e_{\pm} = e_{xx} - e_{yy} \pm 2i e_{xy} + e_{yy}.
\]

In the low-energy the Hamiltonian with wurtzite symmetry one can find in the form

\[
H^{\text{wurt}}_0 = I(\Delta_1 + \Delta_2) + \Delta_y J_y^2 + \Delta_z J_z^2 + (J_x \sigma_x + J_y \sigma_y),
\]

\[
H^{\text{wurt}}_k = A_1 k_x^2 + A_2 k_y^2 + (A_3 k_x^2 + A_4 k_y^2) J_y^2 + A_5 k_x (2[J_y J_z] k_x + 2[J_y J_z] k_y) + A_6 (J_y k_x + J_y k_y) + i A_7 (J_z k_x - J_z k_y),
\]

\[
\frac{\hbar^2}{2m_o}[(A_1 + A_3 J_z^2) k_x^2 + (A_2 + A_4 J_z^2) k_y^2 - A_5 k_y (J_z k_x + J_z k_y)] + i A_7 (J_z k_x - J_z k_y) + (D_1 + D_3 J_z^2) e_{zz} + (D_2 + D_4 J_z^2) e_{z} - D_5 (J_z^2 e_{zz} + J_z^2 e_{z}) - 2D_6 ([J_y J_z] e_{zz} + [J_y J_z] e_{z}),
\]

so we have specified the bases $[Y_{11}, Y_{10}, Y_{1-1}]$ with the operator matrices $J_x, J_y, J_z$. Hence the Effective Hamiltonian can be rewritten in the invariant method like in the article [17]

\[
H = (\Delta_1 + \theta) J_z^2 + \Delta_y J_y^2 + \lambda \sigma_x + \lambda \sigma_y - (K^* J_z^2 + K J_z^2) - (H^* [J_y J_z] + H [J_y J_z]),
\]

where

\[
F = \Delta_1 + J_z^2 + \lambda \sigma_x + \lambda \sigma_y - (K^* J_z^2 + K J_z^2),
\]

\[
G = \Delta_2 + \Delta_y J_y^2 + \lambda \sigma_x + \lambda \sigma_y - (K^* J_z^2 + K J_z^2),
\]

\[
\lambda = \frac{\hbar^2}{2m_o}[(A_1 + A_3 J_z^2) k_x^2 + (A_2 + A_4 J_z^2) k_y^2 - A_5 k_y (J_z k_x + J_z k_y)] + i A_7 (J_z k_x - J_z k_y) + (D_1 + D_3 J_z^2) e_{zz} + (D_2 + D_4 J_z^2) e_{z} - D_5 (J_z^2 e_{zz} + J_z^2 e_{z}) - 2D_6 ([J_y J_z] e_{zz} + [J_y J_z] e_{z}),
\]

\[
\Delta = \sqrt{2}\Delta_3, 
\]

\[
e_{\pm} = e_{zz} \pm 2i e_{xy} + e_{yy},
\]

\[
e_{\pm z} = e_{zz} \pm i e_{yy},
\]

In the article [17] the following expressions are shown to be related with the cubic approximation for effective mass parameters and deformation parameters
\[ A_1 - A_2 = -A_3 = 2A_4, \]
\[ A_3 + 4A_5 = \sqrt{2}A_6, \]
\[ \Delta_2 = \Delta_3, \]
\[ D_1 - D_2 = D_3 = 2D_4, \]
\[ D_3 + 4D_5 = \sqrt{2}D_6. \]

In the article [17] the following basis
\[ [Y_{11} \uparrow], Y_{10} \uparrow, Y_{11} \downarrow, Y_{10} \downarrow, Y_{10} \downarrow, Y_{11} \downarrow, Y_{11} \downarrow \],
are shown to be found the Effective Hamiltonian in the form
\[
H = \begin{pmatrix}
F -H^* -K^* & 0 & 0 & 0 \\
-H & \lambda & H^* & \Delta & 0 & 0 \\
-K & H & G & 0 & \Delta & 0 \\
0 & \Delta & 0 & G & -H^* & -K^* \\
0 & 0 & \Delta & -H & \lambda & H^* \\
0 & 0 & 0 & -K & H & F
\end{pmatrix},
\]
\[ |Y_{11} \uparrow \rangle, |Y_{10} \uparrow \rangle, |Y_{10} \downarrow \rangle, |Y_{11} \downarrow \rangle, |Y_{11} \downarrow \rangle, |Y_{11} \downarrow \rangle, |Y_{11} \downarrow \rangle, |Y_{11} \downarrow \rangle \] (13)

where
\[
K = K_t \exp 2\phi, \\
K_t = \frac{k^2}{2m}A_0 k^2, \\
H = H_t \exp \i \phi, \\
H_t = \frac{k^2}{2m}A_0 k_t k_z, \\
(k_z + \i k_x) = k_t \exp \i \phi, \\
(k_x^2 + k_y^2) = k_t^2.
\]

By introducing the bases \[[\exp -\i 3\phi/2|Y_{11} \uparrow \rangle, \exp \i \phi/2|Y_{11} \downarrow \rangle, \exp -\i \phi/2|Y_{10} \uparrow \rangle, \exp \i 3\phi/2|Y_{10} \downarrow \rangle, \exp -\i \phi/2|Y_{10} \downarrow \rangle, \exp \i \phi/2|Y_{10} \downarrow \rangle]\] and by using the basis transformation
\[
T = \begin{pmatrix}
\alpha^* & 0 & 0 & \alpha & 0 & 0 \\
0 & \beta & 0 & 0 & \beta^* & 0 \\
0 & 0 & \beta^* & 0 & 0 & \beta \\
\alpha^* & 0 & 0 & -\alpha & 0 & 0 \\
0 & \beta & 0 & 0 & -\beta^* & 0 \\
0 & 0 & -\beta^* & 0 & 0 & \beta
\end{pmatrix}
\]
\[ \frac{1}{\sqrt{2}} \exp \i \frac{3\phi}{4} + \frac{\phi}{2}, \beta = \frac{1}{\sqrt{2}} \exp \i \frac{3\phi}{4} + \frac{\phi}{2}, \]
the block diagonal 3 \times 3 Effective Hamiltonian have been found [17]
\[
H' = UHU^* = T^*HT^*,
\]
in the following form
\[
H = \begin{pmatrix}
F & K_t & -\i H_t & 0 & 0 & 0 \\
K_t & G & \Delta - \i H_t & 0 & 0 & 0 \\
\i H_t & \Delta + \i H_t & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & F & K_t & \i H_t \\
0 & 0 & 0 & K_t & G & \Delta + \i H_t \\
0 & 0 & 0 & -\i H_t & \Delta - \i H_t & \lambda
\end{pmatrix}
\]
\[ |\nu \zeta, k_t \rangle = \sum_{i=1}^{3m} |\Psi_k^{(1)}[i, \nu] \psi_i|Z \rangle |1, \zeta, \rangle, \sum_{i=1}^{3m} |\Psi_k^{(2)}[i, \nu] \psi_i|Z \rangle |2, \zeta, \rangle, \sum_{i=1}^{3m} |\Psi_k^{(3)}[i, \nu] \psi_i|Z \rangle |3, \zeta, \rangle. \]

The Bloch vector of \( \nu \)-type hole with spin \( \zeta_\nu = \pm \frac{1}{2} \) and momentum \( k_t \) is specified by its three coordinates \([\Psi_k^{(1)}[m, \nu], \Psi_k^{(2)}[m, \nu], \Psi_k^{(3)}[m, \nu]]\) in the basis \([1, \zeta_\nu], [2, \zeta_\nu], [3, \zeta_\nu]\) [17], known as spherical harmonics with the orbital angular momentum \( l = 1 \) and the eigenvalue \( m \) its \( z \) component. The envelope \( Z \)-dependent part of the quantum well
Figure 1. (Color online) Upper band and Light band for bulk GaN materials.

eigenfunctions can be specified from the boundary conditions \( \psi_m(Z = 0) = \psi_m(Z = 1) = 0 \) of the infinite quantum well as

\[
\psi_m(Z) = \sqrt{\frac{2}{w}} \sin(\pi m Z),
\]

(21)

where \( Z = (\frac{z}{w} + \frac{1}{2}) \), \( m \) is a natural number. Thus the hole wave function can be written as

\[
\Psi_{\nu, k_1}(r) = \frac{e^{ik_1 r}}{\sqrt{A}} |\nu_\nu, k_1\rangle.
\]

(22)

The valence subband structure \( E^\nu_{\nu}(k_1) \) can be determined by solving equations system:

\[
\sum_{j=1}^{3} (H^\nu_{ij}(k_z) = -i \frac{\partial}{\partial z} + V(z) + \delta_{ij}E^\nu_{\nu}(k_1)) \times \phi^\nu_{j \nu}(z, k_z) = 0,
\]

(23)

where \( \phi^\nu_{j \nu}(z, k_z) = \sum_{n=1}^{m} \psi^\nu_{j \nu}[n, \nu] \psi_n(z), i = 1, 2, 3 \). The solutions of Schrödinger equations can be found in the form (20) by premultiply the equations system on (23) functions and by integrating the later equations system on quantum well boundaries.

Let us known the following integral relations for the found solutions algebra equation systems

\[
\int_{0}^{1} \psi(n, Z) \psi(n, Z) dZ = 1,
\]

(24)

if \( n \neq k \),

\[
\int_{0}^{1} \psi(n, Z) \psi(k, Z) dZ = 0,
\]

(25)

if \( n \neq k \),

\[
\int_{0}^{1} \psi(n, Z) \frac{d}{dZ} \psi(k, Z) dZ = -\frac{k((-1)^{n+k}+(1)^{n+k})-2n}{(n+k)(n-k)w},
\]

(26)

if \( n \neq k \),

\[
\int_{0}^{1} \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = 0,
\]

(27)

\[
\int_{0}^{1} \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = -\frac{n^2}{w},
\]

(28)

\[
= -\frac{1}{2} \sum_{n=1}^{m} \frac{1}{\pi m (-n+k) (n+k)} (-4nk(-1)^{-n+k} - 4nk(-1)^{n+k} + 8nk + 2(-1)^{n+k} (-1)^{n+k} (n^2 + k^2)),
\]

(29)

if \( n \neq k \),

\[
\int_{0}^{1} \psi(n, Z) \psi(n, Z) dZ = 0.
\]

(30)
2 Momentum matrix elements of interband phototransitions of GaN/AlGaN Quantum well

Consider a perfect crystal acted upon by a monochromatic plane wave of wave vector \( k \) and frequency \( \omega \). The vector potential of electrical field can be written as follows

\[
A(r, t) = eA_0 \exp(i k r - \omega t). \tag{31}
\]

Hence the electric field vector is taken as

\[
E = \frac{\omega}{c} A. \tag{32}
\]

The magnetic field associated with a plane wave is then given by the following Maxwell equation

\[
\nabla A = 0. \tag{33}
\]

The light and matter interaction Hamiltonian as usually we have written as follows

\[
\hat{V}_{\text{int}} = \frac{\omega}{mc} (A \hat{p} + \frac{\omega}{c} A^2), \tag{34}
\]

where momentum operator is written a like

\[
\hat{p} = -i\hbar \nabla. \tag{35}
\]

Hence the matrix elements of the light and matter interaction Hamiltonian with the considered a perfect crystal acted upon by a monochromatic plane wave of selected vector potential we have described as

\[
\langle \psi_{j' \sigma' k'} | \hat{V}_{\text{int}} | \psi_{j \sigma k} \rangle = \delta_{k', k + \frac{\epsilon_{k'}}{mc}} \int d^3r U_{j' \sigma' k'} [\epsilon \hat{p} + \hbar k \epsilon] U_{j \sigma k}, \tag{36}
\]

where we have written momentum matrix elements for interbands phototransitions as

\[
M_{j \sigma \rightarrow j' \sigma'}(k) = \int d^3r U_{j' \sigma' k} \epsilon \hat{p} U_{j \sigma k}, \tag{37}
\]

as well as

\[
M_{j \sigma \rightarrow \sigma}(k) = \langle \sigma | \epsilon \hat{p} | \sigma \rangle, \tag{38}
\]

<table>
<thead>
<tr>
<th>( \Gamma_1 \times (\Gamma_1 + \Gamma_5) )</th>
<th>( E )</th>
<th>( C_2 )</th>
<th>( 2C_3 )</th>
<th>( 3C_6 )</th>
<th>( 3\sigma_v )</th>
<th>( 3\sigma_v' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi_{j', \mu'}^\nu(g) \chi_{j, \nu}^\nu(g) )</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
| \( \chi_{j, \nu}^\nu(g) \chi_{j', \nu}^\nu(g) \) | (\( A_1 + E_1 \) \times (\( A_1 + E_1 \)) = \( A_1 \times A_1 + E_1 + E_1 \times E_1 \),

Hence the momentum matrix elements of interband phototransitions of bulk GaN have allowed phototransitions because they transform based vector irreducible representations.

The matrix elements \( f_{kq} \) components of irreducible tensor we have derived by Klebs-Gordon coefficients as

\[
\langle n' j' m'| f_{kq} | n j m \rangle = \epsilon^k (-1)^{j_0 - m_0} \frac{j'}{j} \frac{k}{m} \frac{q}{m} \frac{n'}{n} (n' j' m'| f_{kq} | n j m). \tag{40}
\]

The matrix elements \( f_{kq} \) components of irreducible tensor of \( k \) rank have nonzero values for the transitions \( j m \rightarrow j' m' \) which allow by the momentum composition rules \( j' = j + k, \ m' = m + q \).

For irreducible representation of \( C_{6v} \) point group one can inserting the basis

\[
\Gamma_1 + \Gamma_5 : \ Y_{1, \pm 1} = |1, \pm 1 \rangle, \ Y_{10} = |10 \rangle. \tag{41}
\]

The allowed matrix elements of momentum operator can express by using Klebs-Gordon coefficients

\[
\hat{p} : \langle L', m'| \hat{p} | L, m \rangle, \ L = 1, \ L' = 0, \tag{42}
\]
The momentum matrix elements of interband phototransitions of bulk GaN.

Hence one can find
\[ (S|\hat{p}_{\pm}|0) = P_{\sigma}, \quad (S|\hat{p}_{\pm}|1, -1) = - (S|\hat{p}_{\pm}|11) = \sqrt{2} P_{\sigma}. \]

In spherical coordinate axis the electric field vector has the form \( \mathbf{e} = (\sin(\theta) \cos(\varphi), \sin(\theta) \sin(\varphi), \cos(\theta)) \). The total momentum matrix elements can write as follows

\[ M_{\sigma\tau, \sigma\sigma}(k) = \sum_{i=1}^{3} \psi_{ik}^{(0)} \langle S|e_\sigma|v_i, \sigma_\tau \rangle. \]

In the basis
\[
\begin{align*}
|1, \sigma_\tau \rangle &= \frac{1}{\sqrt{2}} (Y_1^1 \psi(-1/2)e^{-i\varphi/2}e^{-3i\pi/4} \pm Y_1^{-1} \psi(-1/2)e^{3i\varphi/2}e^{3i\pi/4}), \\
|2, \sigma_\tau \rangle &= \frac{1}{\sqrt{2}} (\pm Y_1^1 \psi(1/2)e^{-i\varphi/2}e^{-i\pi/4} + Y_1^{-1} \psi(1/2)e^{i\varphi/2}e^{i\pi/4}), \\
|3, \sigma_\tau \rangle &= \frac{1}{\sqrt{2}} (\pm Y_0^0 \psi(1/2)e^{-i\varphi/2}e^{-i\pi/4} + Y_0^0 \psi(-1/2)e^{i\varphi/2}e^{i\pi/4}).
\end{align*}
\]

the Hamiltonian may be transformed to the diagonal form indicating two spin degeneracy \[ \text{[17]}: \]
\[ H_\pm = \begin{pmatrix} F & K_{t} & \mp iH_{t} \\ K_{t} & G & \Delta \mp iH_{t} \\ \pm iH_{t} & \Delta \mp iH_{t} & \lambda \end{pmatrix} \begin{pmatrix} 1, \zeta_\tau \rangle \\ 2, \zeta_\tau \rangle \\ 3, \zeta_\tau \rangle \end{pmatrix}. \]

By using matrix algebra calculations one can find
\[ \langle S|e_\sigma|v_i, \sigma_\tau \rangle: \]

\[ (S|\uparrow \uparrow|v_1, \pm) = (S|\uparrow \uparrow|1 \frac{1}{\sqrt{2}} \sin \theta (\exp(i\varphi) \hat{p}_\pm + \exp(-i\varphi) \hat{p}_\pm) + \hat{p}_\pm \cos \theta | = - \frac{1}{2} P_\sigma \exp(i\varphi) \exp(-i \frac{3\pi}{4}) \sin \theta, \]

\[ (S|\uparrow \downarrow|v_2, \pm) = (S|\uparrow \downarrow|1 \frac{1}{\sqrt{2}} \sin \theta (\exp(i\varphi) \hat{p}_\pm + \exp(-i\varphi) \hat{p}_\pm) + \hat{p}_\pm \cos \theta | = \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \sin \theta, \]

\[ (S|\downarrow \uparrow|v_3, \pm) = (S|\downarrow \uparrow|1 \frac{1}{\sqrt{2}} \sin \theta (\exp(i\varphi) \hat{p}_\pm + \exp(-i\varphi) \hat{p}_\pm) + \hat{p}_\pm \cos \theta | = \pm \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \cos \theta, \]

\[ (S|\downarrow \downarrow|v_4, \pm) = (S|\downarrow \downarrow|1 \frac{1}{\sqrt{2}} \sin \theta (\exp(i\varphi) \hat{p}_\pm + \exp(-i\varphi) \hat{p}_\pm) + \hat{p}_\pm \cos \theta | = \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \cos \theta, \]

By using matrix algebra calculations one can find
\[ \langle S|e_\sigma|v_i, \sigma_\tau \rangle: \]

| \( |v_1, \pm \rangle \) | \( |v_2, \pm \rangle \) | \( |v_3, \pm \rangle \) |
|---|---|---|
| \( (S|\uparrow \uparrow|v_1, \pm) = - \frac{1}{2} P_\sigma \exp(i\varphi) \exp(-i \frac{3\pi}{4}) \sin \theta \) | \( \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \sin \theta \) | \( \pm \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \cos \theta \) |
| \( (S|\downarrow \downarrow|v_1, \pm) = \frac{1}{2} P_\sigma \exp(-i\varphi) \exp(i \frac{3\pi}{4}) \sin \theta \) | \( \pm \frac{1}{2} P_\sigma \exp(i\varphi) \exp(-i \frac{3\pi}{4}) \sin \theta \) | \( \frac{1}{2} P_\sigma \exp(i\varphi) \exp(-i \frac{3\pi}{4}) \cos \theta \) |
3 Quantum Hall plateaus of Gain coefficients of GaN/AlGaN Quantum well including Lorenz broadening

It is known [18], that the optical material gain can be calculated from Fermi golden rule:

$$\alpha_0 = \frac{\pi e^2}{\epsilon \sqrt{m_0 \omega}} \sum_{\alpha, \beta=1, \uparrow \downarrow} \sum_{\sigma, =+,-} \int \phi \, d\phi \frac{1}{2\pi} \left| e M_{m\alpha}^{\sigma, \sigma'}(k_i) \right|^2 (f_m^{\sigma'}(k_i) - f_{\sigma, a}(k_i)) \delta(E_{\sigma, ma}(k_i) - \hbar \omega),$$  (54)

where $e$ is the magnitude of the electron charge, $m_0$ is the electron rest mass in free space, $c$ is the velocity of light in free space, $\epsilon$ is the permittivity of the host material, $f_m^{\sigma'}$, $f_{\sigma, a}$ are the Fermi-Dirac distributions for electrons in the conduction and valence bands, $e$ is a unit vector of vector potential of electromagnetic field, $E_{\sigma, ma}(k_i)$ is the interband energy of the conduction and valence bands, and $\hbar \omega$ is an optical energy. We consider the electromagnetic wave, which propagates in plane of quantum well.

Although the carriers within each band are in a strongly nonequilibrium states, however interband relaxation times are much larger than intraband relaxation times. Therefore the Fermi-Dirac statistics may be used in the calculations.

$$M_{m\alpha}^{\sigma, \sigma'}(k_i) = \langle \Psi_{m, k_i}^{\alpha, \sigma} | \hat{p} | \Psi_{m, k_i}^{\alpha, \sigma'} \rangle$$

is the momentum matrix elements for transitions between the conduction band state $\Psi_{m, k_i}^{\alpha, \sigma}(x)$ and the valence band state $\Psi_{m, k_i}^{\alpha, \sigma'}(x)$, and $\hat{p}$ is momentum operator.

The optical material gain [18, 19] can be calculated from Fermi golden rule:

$$\alpha_0 = \frac{\pi e^2}{\epsilon \sqrt{m_0 \omega}} \sum_{\alpha, \beta=1, \uparrow \downarrow} \sum_{\sigma, =+,-} \int \phi \, d\phi \frac{1}{2\pi} \left| e M_{m\alpha}^{\sigma, \sigma'}(k_i) \right|^2 \frac{\left(f_m^{\sigma'}(k_i) - f_{\sigma, a}(k_i)\right)(\frac{\hbar \omega}{\gamma})}{(E_{\sigma, ma}(k_i) - \hbar \omega)^2 + \gamma^2},$$  (55)

where $e$ is the magnitude of the electron charge, $m_0$ is the electron rest mass in free space, $c$ is the velocity of light in free space, $\kappa = 8.27$ is the permittivity of the host material, $f_m^{\sigma'}$, $f_{\sigma, a}$ are the Fermi-Dirac distributions for electrons in the conduction and valence bands, $e$ is a unit vector of vector potential of electromagnetic field, $E_{\sigma, ma}(k_i)$ is the interband energy of the conduction and valence bands, and $\hbar \omega$ is an optical energy, $\hbar \gamma$ is a half linewidth of the Lorentzian functions, which is equal 6.56 meV.

In the article in Figs.2,3 the considerable plateaus in the Gain coefficients calculations at interband phototransitions without including Lorenz broadening are shown to be related with Quantum Spin Hall effects in intraband phototransitions of GaN/AlGaN Quantum well.
Quantum Spin Hall effect in intersubband transitions of GaN/AlGaN Quantum well coefficients and integrating (63) on quantum well boundaries. The solutions of Schrödinger equations system can seek in the form (62) by premultiply the integral (63) on (21)

We take the following plain wave functions written as vectors in the three-dimensional Bloch space:

$$\psi_{j\sigma k}(r) = \exp(i kr)\psi_{j\sigma k}(r),$$

$$\psi_{j\sigma k}(r) = (r|j\sigma k) + O(k),$$

$$|j\sigma k) = \phi_{j\sigma k}^{(1)}|\phi_1, \sigma) + \phi_{j\sigma k}^{(2)}|\phi_2, \sigma) + \phi_{j\sigma k}^{(3)}|\phi_3, \sigma) = |\phi_{j\sigma k}) = \phi_{j\sigma k},$$

$$|\phi_{j\sigma k}) = [|\phi_1, \sigma), |\phi_2, \sigma), |\phi_3, \sigma),$$

$$\varphi = \varphi_k = \arctan \frac{k}{k_z}.$$

From the conditions of orthonormal basis of wave plane vectors if $k_z = 0$ then $\phi_{jk+}=\phi_{jk-}=\phi_{jk}$, $|\phi_{jk}|^2 = 1 \Rightarrow$ Hence

$$\phi_{jk} = \begin{bmatrix} \phi_{j\sigma k}^{(1)} \\ \phi_{j\sigma k}^{(2)} \\ \phi_{j\sigma k}^{(3)} \end{bmatrix} = \begin{bmatrix} \cos \theta_{jk} \\ \cos \phi_{jk} \sin \theta_{jk} \\ \sin \phi_{jk} \sin \theta_{jk} \end{bmatrix}$$

The solutions of Schrödinger equations system can seek in the form (62) by premultiply the integral (63) on (21) functions and integrating (63) on quantum well boundaries

$$|j\varsigma r k_z) = \begin{bmatrix} \sum_{i=1}^{m} \cos \theta_{i,j,k} \psi_i(Z) \\ \sum_{i=1}^{m} \sin \phi_{i,j,k} \sin \theta_{i,j,k} \psi_i(Z) \end{bmatrix} |1, \varsigma_r),$$

$$[\nabla (v_1, \sigma', v_{1\kappa}) + \nabla (v_2, \sigma', v_{2\kappa}) + \nabla (v_3, \sigma', v_{3\kappa})] \left( \frac{\partial H_z}{\partial \psi} (\sin \theta \cos \varphi) + \frac{\partial H_z}{\partial \psi} (\sin \theta \sin \varphi) + \frac{\partial H_z}{\partial \psi} (\cos \varphi) \right) = 0$$

$$= [\nabla (v_{1\kappa} \sigma_1, \sigma_0) + \nabla (v_{2\kappa} \sigma_1, \sigma_0) + \nabla (v_{3\kappa} \sigma_1, \sigma_0)] = \hbar \sum_{i,i'=1}^{3} (\delta_{\sigma',\sigma} [K_{\sigma,i}^{(k)} \cos \theta + K_{\sigma,i}^{(k)} \sin \theta \cos \varphi] + K_{\sigma,i}^{(k)} \sin \theta \sin \varphi)]_{j\kappa | j\kappa},$$

Figure 3. (Color online) Gain coefficient for the quantum well GaN/Al$_0.3$Ga$_0.7$N with a width 26 Å, at a carriers concentration $10^{13}$ cm$^{-2}$, at a temperature 4.4 K, 77 K and 300 K including Lorentz broadening.

4 Quantum Spin Hall effect in intersubband transitions of GaN/AlGaN Quantum well without Landau levels

We take the following plain wave functions written as vectors in the three-dimensional Bloch space:

$$\psi_{j\sigma k}(r) = \exp(i kr)\psi_{j\sigma k}(r),$$

where $j = HH, LH, SH,$

$$\psi_{j\sigma k}(r) = (r|j\sigma k) + O(k),$$

$$|j\sigma k) = \phi_{j\sigma k}^{(1)}|\phi_1, \sigma) + \phi_{j\sigma k}^{(2)}|\phi_2, \sigma) + \phi_{j\sigma k}^{(3)}|\phi_3, \sigma) = |\phi_{j\sigma k}) = \phi_{j\sigma k},$$

$$|\phi_{j\sigma k}) = [|\phi_1, \sigma), |\phi_2, \sigma), |\phi_3, \sigma),$$

$$\varphi = \varphi_k = \arctan \frac{k}{k_z}.$$

From the conditions of orthonormal basis of wave plane vectors if $k_z = 0$ then $\phi_{jk+}=\phi_{jk-}=\phi_{jk}$, $|\phi_{jk}|^2 = 1 \Rightarrow$ Hence

$$\phi_{jk} = \begin{bmatrix} \phi_{j\sigma k}^{(1)} \\ \phi_{j\sigma k}^{(2)} \\ \phi_{j\sigma k}^{(3)} \end{bmatrix} = \begin{bmatrix} \cos \theta_{jk} \\ \cos \phi_{jk} \sin \theta_{jk} \\ \sin \phi_{jk} \sin \theta_{jk} \end{bmatrix}$$

The solutions of Schrödinger equations system can seek in the form (62) by premultiply the integral (63) on (21) functions and integrating (63) on quantum well boundaries

$$|j\varsigma r k_z) = \begin{bmatrix} \sum_{i=1}^{m} \cos \theta_{i,j,k} \psi_i(Z) \\ \sum_{i=1}^{m} \sin \phi_{i,j,k} \sin \theta_{i,j,k} \psi_i(Z) \end{bmatrix} |1, \varsigma_r),$$

$$[\nabla (v_1, \sigma', v_{1\kappa}) + \nabla (v_2, \sigma', v_{2\kappa}) + \nabla (v_3, \sigma', v_{3\kappa})] \left( \frac{\partial H_z}{\partial \psi} (\sin \theta \cos \varphi) + \frac{\partial H_z}{\partial \psi} (\sin \theta \sin \varphi) + \frac{\partial H_z}{\partial \psi} (\cos \varphi) \right) = 0$$

$$= [\nabla (v_{1\kappa} \sigma_1, \sigma_0) + \nabla (v_{2\kappa} \sigma_1, \sigma_0) + \nabla (v_{3\kappa} \sigma_1, \sigma_0)] = \hbar \sum_{i,i'=1}^{3} (\delta_{\sigma',\sigma} [K_{\sigma,i}^{(k)} \cos \theta + K_{\sigma,i}^{(k)} \sin \theta \cos \varphi] + K_{\sigma,i}^{(k)} \sin \theta \sin \varphi)]_{j\kappa | j\kappa},$$

Page 10 of 16
where from conditions of inverted bands

\[
\begin{align*}
\theta_{hh} &= 0, \\
\theta_{lh,sh} &= \frac{\pi}{2}, \\
\phi_{th} &= \phi_{sh} - \frac{\pi}{2}.
\end{align*}
\]

In the basis

\[
|1, \zeta\rangle = \frac{1}{\sqrt{3!}} (Y_1^1 \psi(1/2) e^{-3i\varphi/2} e^{-3i\nu/4} \pm Y_1^{-1} \psi(-1/2) e^{3i\nu/2} e^{3i\varphi/4} + Y_1^0 \psi(0) e^{i\varphi/2} e^{i\varphi/4})
\]

\[
|2, \zeta\rangle = \frac{1}{\sqrt{3!}} (Y_1^1 \psi(-1/2) e^{-i\varphi/2} e^{-i\nu/4} + Y_1^{-1} \psi(1/2) e^{i\varphi/2} e^{i\nu/4} + Y_1^0 \psi(0) e^{-i\varphi/2} e^{-i\nu/4})
\]

\[
|3, \zeta\rangle = \frac{1}{\sqrt{3!}} (Y_1^1 \psi(1/2) e^{-i\varphi/2} e^{-i\nu/4} + Y_1^{-1} \psi(-1/2) e^{i\varphi/2} e^{i\nu/4} + Y_1^0 \psi(0) e^{i\varphi/2} e^{i\nu/4})
\]

the Hamiltonian may be transformed to the diagonal form indicating two spin degeneracy [17]:

\[
H_{\pm} = \begin{pmatrix} F & K_t & \mp iH_t \\ K_t & G & \mp iH_t \\ \pm iH_t & \Delta \mp iH_t & \lambda \end{pmatrix}
\]

\[
|1, \zeta\rangle, |2, \zeta\rangle, |3, \zeta\rangle.
\]

We have considered intersubband phototransitions because the influence of time inversion operator has been importantly inserted [20]:

\[
K^{(t, t)} = -\frac{m_0}{\hbar k_t} \delta H_k, \quad K^{(t, \nu)} = -\frac{m_0}{\hbar k_t} \{H, \frac{dU}{d\nu} U^{-\nu}\}.
\]

Quantum Hall effect is connected with intersubband transitions of bulk GaN. Quantum Hall effect is specified as spin Hall effect. The intrinsic Hall conductivity can write as follows [21]

\[
\sigma^{int} = \frac{e^2}{h} \int d^3 k f(k) \Omega(k),
\]

\[
\sigma^{int} = \frac{e^2}{h} \int d^2 k f(k) \Omega(k),
\]

where three dimensional space can be specified as follows

\[
\frac{1}{V} \sum_{k} \rightarrow \frac{1}{V} \frac{\nu}{(2\pi)^2} \int d^3 k = \frac{1}{(2\pi)^2} 2\pi \int k_t d k_t d k_z,
\]

where in planar can be specified space

\[
d^2 k = \frac{dk}{(2\pi)^2},
\]

\[
\frac{1}{\hbar} \sum_{k_t} \rightarrow \frac{1}{\hbar} \frac{\nu}{(2\pi)^2} \int d^2 k_t = \frac{1}{2\pi^2} 2\pi \int k_t d k_t,
\]

\[
f(k) \text{ is Fermi-Dirac distribution function. The generic form of Berry curvature is defined by expression [21]}
\]

\[
\Omega_j(k) = \hat{z} \cdot \nabla_k \times \langle v_j(k) | \nabla_k | v_j(k) \rangle,
\]

where $|v_j(k)\rangle$ is the periodic part of the Bloch function and $j$ is the band index. The Berry curvature [21] of bulk GaN which are shown to be connected with intraband phototransitions with $G_{0 \nu}$ point group symmetry is specified like

\[
\Omega_j(k) = \hat{z} \det \begin{pmatrix} e_z & e_z & e_z \\ -m_0 \frac{\partial H_k}{\partial k_z} & m_0 \frac{\partial H_k}{\partial k_z} & -m_0 \frac{\partial H_k}{\partial k_z} \\ 0 & 0 & \left[ \frac{\partial U}{\partial \nu} \right] \end{pmatrix},
\]
The irreducible representation of $C_{6v}$ [15].

<table>
<thead>
<tr>
<th>$C_{6v}$</th>
<th>$\Gamma_1 + \Gamma_5$</th>
<th>$E$</th>
<th>$C_2$</th>
<th>$2C_3$</th>
<th>$2C_6$</th>
<th>$3\sigma_v$</th>
<th>$3\sigma'_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^+(g)$</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\chi^-(g)$</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}[+]$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>$2\Gamma_1 + \Gamma_5 + \Gamma_6$</td>
</tr>
<tr>
<td>$\frac{1}{2}[-]$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>$\Gamma_2 + \Gamma_5$</td>
</tr>
</tbody>
</table>

Table 8. The irreducible representation of $C_{6v}$ [15].

<table>
<thead>
<tr>
<th>$C_{6v}$</th>
<th>$E$</th>
<th>$C_2$</th>
<th>$2C_3$</th>
<th>$2C_6$</th>
<th>$3\sigma_v$</th>
<th>$3\sigma'_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_v$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{1}{2}[+]$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\frac{1}{2}[-]$</td>
<td>3</td>
<td>-1</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
</tr>
</tbody>
</table>

5 Quantum Spin Hall effect in intersubband transitions of bulk GaN

Unlike for the transitions between the valence and conduction band, the Bloch function contributions of zeroth order in $k$ vanish due to symmetry properties (except the small part proportional to the spin splitting constant $k_j$). Because Intraband phototransitions integral can write in the generic form [20, 22, 23]

$$ (U_{j\sigma'k}|p|U_{j\sigma k}) : $$

$$ \chi_r(g)\frac{1}{2}[(\chi_r^{(v)}(g) - \chi_r^{(s)}(g))] = (A_1 + E_1) \times (A_2 + E_1) = (E_1 \times E_1). $$

But we consider intersubband phototransitions because the influence of time inversion operator has been importantly inserted [20]. So

$$ \langle j'\sigma'k|e^{\frac{\theta H}{\hbar k}}|j\sigma k \rangle : $$

$$ \chi_{\mu}^{(s)}(g)\chi_r(g)\chi_{\nu}^{(s)}(g) = (A_1 + E_1) \times (A_1 + E_1) = (A_1 \times A_1) + (E_1 \times E_1). $$

As well known vector can be transformable by vector representation $(A_1 \times A_1) + (E_1 \times E_1)$. In the paper the expressions for momentum matrix elements have presented in the following general form:

$$ o\langle j'\sigma'k|e^{\frac{\theta H}{\hbar k}}|j\sigma k \rangle_0, $$

$$ |j\sigma k \rangle_\varphi = U_\varphi|j\sigma k \rangle_0. $$

One can derive the expressions for calculation of momentum matrix elements for interband transitions as well as intraband transitions in stimulated emission explaining

$$ [o\langle v_1, \sigma'^{(3)}_{j'k\alpha'\varphi} + 0\langle v_2, \sigma'^{(2)}_{j'k\alpha'\varphi} + 0\langle v_3, \sigma'^{(3)}_{j'k\alpha'\varphi} + U_{j\sigma k} e^{\frac{\theta H}{\hbar k}} U_{-\varphi}|v_{j\sigma' k}, \sigma' \rangle_0 + v_{j\sigma' k}, \sigma' \rangle_0 + v_{j\sigma' k}, \sigma' \rangle_0]. $$

Since

$$ \frac{\partial}{\partial k}(U_{j\varphi}H_{U_{-\varphi}}) = U_{j\varphi}H_{U_{-\varphi}} \frac{\partial U_{j\varphi}}{\partial k} U_{-\varphi} + d\varphi \frac{\partial}{\partial k} U_{j\varphi} U_{-\varphi} + U_{j\varphi} \frac{\partial}{\partial k} U_{-\varphi} + \frac{\partial}{\partial k} U_{-\varphi} = $$

$$ = H_{U_{j\varphi}} \frac{\partial U_{j\varphi}}{\partial k} U_{-\varphi} + U_{j\varphi} \frac{\partial}{\partial k} U_{-\varphi} + U_{j\varphi} \frac{\partial}{\partial k} H, $$

as well as using

$$ U_{j\varphi} U_{-\varphi} = 1 \Longrightarrow \frac{d}{d\varphi}(U_{j\varphi} U_{-\varphi}) = 0 \Longrightarrow U_{j\varphi} \frac{dU_{j\varphi}}{d\varphi} U_{-\varphi} + \frac{dU_{j\varphi}}{d\varphi} U_{-\varphi} = 0, $$

one can find the following unitary transformation for momentum

$$ \frac{\partial}{\partial k}(U_{j\varphi}H_{U_{-\varphi}}) = -H_{U_{j\varphi}} U_{j\varphi} U_{-\varphi} + U_{j\varphi} \frac{\partial}{\partial k} U_{-\varphi} + U_{j\varphi} \frac{\partial}{\partial k} H, $$

$$ (83) $$

$$ (84) $$

$$ (85) $$
In the basis of spherical wave functions with the orbital angular momentum \( F \)

In the low-energy limit the Hamiltonian of wurtzite

In the spherical coordinate system one can find

Therefore a formula (82) can be rewritten as following

where

In the low-energy limit the Hamiltonian of wurtzite

In the basis of spherical wave functions with the orbital angular momentum \( l = 1 \) and the eigenvalue \( m_i \) of its \( z \) component:

the Hamiltonian may be transformed to the diagonal form indicating two spin degeneracy \([17]:\)
Inserting the new basis [24, 25]

\[ |v_1, \pm \rangle = \frac{1}{\sqrt{2}}(|1, 1\rangle \pm |1, -1\rangle) \exp^{-i\varphi^2/2},
|v_2, \pm \rangle = \frac{1}{\sqrt{2}}(|\pm 1, 0\rangle \pm |1, -1\rangle) \exp^{-i\varphi^2/2},
|v_3, \pm \rangle = \frac{1}{\sqrt{2}}(|\pm 1, 0\rangle \pm |1, 0\rangle) \exp^{-i\varphi^2/2} \]

one can find the basis [24, 25] from the derivatives of Hamiltonian built on which have derived a matrix formulas of (83) and (84)

\[ |u_1, \pm \rangle = |v_1, \pm \rangle,
|u_2, \pm \rangle = \frac{1}{\sqrt{2}}(|v_2, \pm \rangle - \frac{1}{\sqrt{3}}|v_3, \pm \rangle),
|u_3, \pm \rangle = \frac{1}{\sqrt{2}}(|v_2, \pm \rangle + \frac{1}{\sqrt{3}}|v_3, \pm \rangle) \]

Also keeping in mind the spherical coordinate system and from conditions of inverted bands wave vectors solutions of Schrödinger equations one can find

\[ \nu_{jk} = \begin{bmatrix} \cos \theta_{jk} \\ \sin \theta_{jk} \end{bmatrix}, \theta_{hh} = 0, \theta_{hh} = \frac{\pi}{2}, \phi_{lh} = \phi_{hh} - \frac{\pi}{2}, \]

after the solving the integral (88) the intrasubband phototransitions have found as following

\[ \left| \frac{\sum_{j} |E_{j-\text{hh}}|^2 / \hbar^2}{\sum_{j} |E_{j-\text{hh}}|^2 / \hbar^2} \right| = (9\gamma_2^2 k_z^2 + 2k_z^2)(\sin \phi)^2 \right| (\cos \phi)^2 \right| (\sin \theta)^2 \right|,
\]

as well as

\[ \left| \frac{\sum_{j} |E_{j-\text{hh}}|^2 / \hbar^2}{\sum_{j} |E_{j-\text{hh}}|^2 / \hbar^2} \right| = (9\gamma_2^2 k_z^2 + 2k_z^2)(\cos 2\phi)^2(\sin \theta)^2 + (\gamma_{12} - 2\gamma_{23})k_z^2 + (\gamma_{12} + 4\gamma_{33})k_z^2(\sin 2\phi)^2(\cos \theta)^2, \]

Quantum Hall effect is connected with intersubband transitions of bulk GaN. Quantum Hall effect is specified as spin Hall effect [26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. The intrinsic Hall conductivity can write as follows [21]

\[ \sigma^{int} = \frac{e^2}{h} \int d^2k f(k)\Omega(k), \]

where in planar specified space

\[ d^2k = \frac{dk}{(2\pi)^2}, \]

\( f(k) \) is Fermi-Dirac distribution function. The generic form of Berry curvature is defined by expression [21]

\[ \Omega_j(k) = \hat{z} \cdot \nabla_k \times \{ \nu_j(k) | \nu_j(k) \}, \]

where \( |\nu_j(k)\rangle \) is the periodic part of the Bloch function and \( j \) is the band index. The Berry curvature [21] of bulk GaN which are shown to be connected with intraband phototransitions with \( C_{6\nu} \) point group symmetry is specified like

\[ \Omega_j(k) = \hat{z} \det \begin{bmatrix} e_x & e_- & e_z \\ \hat{K}^{(i)} & \hat{K}^{(v)} \\ 0 & 3\gamma_z k_z \sin (\phi) \sin (\theta) \end{bmatrix} = e_x \hat{K}^{(i)} 3\gamma_z k_z \sin (\phi) \sin (\theta) - e_- \hat{K}^{(v)} 3\gamma_z k_z \sin (\phi) \sin (\theta). \]

Since Theta function of Hevithaide is specified like

\[ \theta(x) = \lim_{k \to -\infty} \frac{1}{1 + \exp(-2kx)}, \]

then \( f(k) = \frac{1}{1 + \exp(\theta(xk))} \) is Fermi-Dirac distribution function at the temperature \( T = 4.4K \).
Datasets for the valence band effective mass parameters $A_i$, where $i = 1..6$, for the deformation potentials $D_i$, where $i = 1..4$ in meV and for the energy parameters $\Delta_1 = \Delta_\epsilon$, as well as $\Delta_4 = \Delta_\mu/3$ in meV for bulk GaN materials, and elastic constants $C_{ij}$ in GPa.

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
<th>$A_6$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$E_g$</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
<th>$C_{13}$</th>
<th>$C_{33}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.56</td>
<td>-0.91</td>
<td>5.65</td>
<td>-2.83</td>
<td>-3.13</td>
<td>-4.86</td>
<td>700</td>
<td>2100</td>
<td>1400</td>
<td>-700</td>
<td>3507</td>
<td>16</td>
<td>4</td>
<td>106</td>
<td>398</td>
</tr>
</tbody>
</table>

Datasets for the concentrations of holes $p$ in cm$^{-3}$, the temperatures $T$ in K, the Fermi energies $E_F^p$ of holes of bulk GaN in meV, the Fermi wave vectors $k_F^p$ of holes in cm$^{-1}$ at the dimensionless wave vector of three dimensional space of bulk GaN $k_0^{3\text{dim}}$ in cm$^{-1}$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$T$</th>
<th>$E_F^p$</th>
<th>$k_F^p$</th>
<th>$k_0^{3\text{dim}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.5 \times 10^{16}$</td>
<td>4.4</td>
<td>23.01</td>
<td>0.15</td>
<td>$1.38 \times 10^4$</td>
</tr>
<tr>
<td>$10^{16}$</td>
<td>4.4</td>
<td>23.27</td>
<td>0.095</td>
<td>$1.38 \times 10^4$</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>4.4</td>
<td>24.335</td>
<td>1.38</td>
<td>$1.38 \times 10^4$</td>
</tr>
</tbody>
</table>

We have found the upper bound for specify definite integral of wave vectors by solving the following equations $-\frac{\epsilon_F + \mu}{2} = 0$. In the table 10 the upper bound are shown to be related with the Fermi wave vectors $k_F^p$ of holes and we have found that for holes of bulk GaN $k_F^{upper\text{bound}} = k_0^{3\text{dim}} = k_F^p$.

References


