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Integer Quantum Hall Effect, phase transition in researches of domain wall for data storage of thin Os$_x$Ra$_{1-x}$FeO$_3$ nanofilms with channel of p-n junction

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Abstract The thin Os$_x$Ra$_{1-x}$FeO$_3$ nanofilms have quasiparticle excitations that can be described by (2+1)-dimensional Dirac theory from developed a four-sublattice model (FSM) for the analytical description of A-cation displacements in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type. We have found that FSM explains the coexistence of rhombohedral (R), orthorhombic (O) and spatially modulated phases observed by atomic-resolution scanning transmission electron microscopy (STEM) in Os-doped RaFeO$_3$. Using this approach we atomically resolve the theoretical model of the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in Os$_x$Ra$_{1-x}$FeO$_3$ polymorphs. We have shown that this produces an unconventional form of the quantized Hall conductivity $\sigma_{xy} = -(2e^2/h)(2n+1)$ with $n = 0, 1, 2, ...$. This unconventional quantization is caused by the analytical chirality of $n = 0$ Landau level. In the article we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize PmBaMn$_2$O$_6$ ferroics with incommensurately modulated sinusoidal polarization waves $Q_T$, $Q_P$, $Q_b$. The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for $Q_T$, $Q_P$, $Q_b$ with sinusoidal depolarization waves for the thin PmBaMn$_2$O$_6$ nanofilm are specified. We have presented the computation solutions of quantized normalized charges densities of the Eq. (96) stationary Schrödinger boundary problem for different thickness $h$ of PmBaMn$_2$O$_6$ ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition in the form of quantized hysteresis loops that were clearly visible like Fairy-tale Phoenix. We have found the quantized out-of-plane polarizations $P_3$ dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilm with semiconductor with different boundary parameters.

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Spin Hall insulators have connected with respect to dissipationless spin transport via a sample with the spin-orbit interactions (SOI) effects. The extremum rings of the valence band in tensile strained zincblende GaN quantum well as well as the two valley shape of the valence band in GaN nanotube grown along c axis were a collection have attracted our attention both for their fundamental studies and for their device applications. The Quantum Spin Hall effects are shown to be related with intraband transitions of bulk GaN. In the framework of the effective mass theories we have solved the Schrödinger equation if the topological insulator transformation is achieved. The exact solutions of the Schrödinger equations as well as Quantum Spin Hall effect of intraband transitions of bulk GaN are found. For the hexagonal symmetry of GaN the Effective Hamiltonian based on $C_{6v}$ point symmetry group was found. In the article for Quantum Hall effect of intraband phototransitions of bulk GaN the expressions of Berry curvature as well as Hall conductivities have been found when the topological insulator transformation is achieved. Spin Hall insulators have connected with the Topological insulator (TI) as well as Exciton insulators. The dissipationless spin transport via the sample are shown to be related with Spin Hall insulator self-consistent solution of the Schrödinger equations for electrons and holes and the Poisson equations at the presence of spatially varying quantum well potential due to the piezoelectric effect and the local exchange-correlation potential in ZnO quantum well. In ZnO/Zn,MgO quantum well the electron-hole pairing leads to the exciton insulator states. An exciton insulator states with a gap 3.4 eV of ZnO quantum well and at 310 K temperature are predicted. If the electron and hole are separated, their energy is higher on 0.2 meV than if they are paired. The particle-hole pairing leads to the Cooper instability.
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2 Introduction

The free energy expansion in powers of order parameters is employed in Landau-Ginsburg-Devonshire (LGD) free energy and can be used in the phase-field modeling of macro and nanosized ferroelectrics [1]. Understanding of ferroic behavior at surfaces, interfaces, and defects as well as the nature of ferroelectric states considerably advanced in the last decades with the advancement of scanning transmission electron microscopy (STEM) [1]. Probing the unit-cell level symmetry breaking via STEM allowed the determination of direct atomic positions, from which the spatial distributions of order parameters fields can be mapped [1]. Hence here we derive a model LGD-type free energy describing directly observable degrees of freedom available from atomic-resolution STEM [1]. We have proposed a theoretical four-sublattice model (FSM) for analytical description of cation displacement in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type, that explain the coexistence of rhombohedral (R), orthorhombic (O), and spatially modulated (SM) phases observed by atomic-resolution STEM [1]. Using this approach we atomically resolve and theoretically model the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in perovskites of Os$_x$Ra$_{1-x}$FeO$_3$ polymorphs with magnetic field. Hence for over 50 years of researches of ferroic materials the properties of these materials are explored using the combination of scattering techniques that provides the information on the nature and symmetry of order parameters and macroscopic measurements that provided the information on the corresponding expansion coefficients and the nature of phase transitions [1].

In this earlier work, the authors [2] studied a fixed filling factor while varying the flux per plaquette from small values and large unit cells, where the standard Landau level description holds, to somewhat larger flux values and smaller unit cells, where that description broke down. As they were able to change this parameter without any evidence of encountering a phase transition, the latter limit constituted an observation of the FQHE in the presence of strong lattice effects [2].

In a landmark paper in 1982, Thouless, Kohmoto, Nightingale, and den Nijs [3] analyzed the uniform-field Hall effect in a strong periodic potential that was known to lead to an intricate spectrum, the so-called Hofstadter butterfly (see
Fig. 2; they showed that it gives rise to an integer QHE under certain conditions, i.e., whenever the chemical potential lies in a gap [2]. Indeed, the Hall conductance was shown to map to a topological invariant associated with filled bands - the (first) Chern number. In this model, time-reversal symmetry is broken by a spatially inhomogeneous magnetic field with zero average, and the Hall conductance again equals the Chern number of the band [2]. Six years later in another striking development Haldane answered the second question showing by an explicit construction of a tight-binding model on a honeycomb lattice that a quantized Hall conductance can arise from a fully filled band even in the absence of a net magnetic field [2].

A Landau level involves a set of exactly degenerate single-particle states and thus, at a fractional filling, the kinetic energy alone does not select a ground state, but instead, it falls to the interactions to force the issue [2]. By contrast, a Chern band typically will have a significant dispersion that will select a unique kinetic-energy-dominated ground state at reasonable interaction strengths, as it does in all metals. Recognizing this, all three paper [4, 5, 6] devote considerable effort to constructing lattice models with nearly flat (degenerate) Chern bands. The authors of article [5] construct a flattened version of Haldane’s model on a square lattice. They note that while a fully flattened model requires the inclusion of electron hopping over arbitrarily large distances, the hopping amplitudes decrease exponentially, which allows a relatively flat band to be constructed by keeping a small set of hopping amplitudes. The relevant flatness parameter, which should be large for the effects of interactions to be important, is the ratio of the band gap (which sets a bound on the strength of the interactions one can safely include) to the bandwidth and they show how to get this number up to seven with just second-neighbor interactions. Similarly, the authors of articles [4, 6] construct models on the kagome and checker-board lattices, which also exhibit large values of the flatness parameters [2].

With a flat Chern band in hand, the authors of the paper [5] introduce interactions and study the system at a fractional filling of 1/3 through numerical computation on a sized system. They find two of the classic signatures of the 1/3 FQHE state: a fractional quantum Hall conductance that was close to the filling fraction, and a nontrivial ground-state degeneracy with periodic boundary conditions. As a test, they vary the band structure continuously to a topologically trivial band and find that these features go away. In a related piece of unpublished work, another group finds similar results at filling of 1/3 and 1/5 [2]. Altogether, this work offers strong evidence that fractionally filled Chern bands do indeed exhibit the FQHE [2].

The unique band spectrum of graphene as well as 2D Weil PlCl3, PlBr3, PdCl3, RuCl3, and Group-VI Dichalcogenides materials leads to the unconventional integer quantum Hall effect (IQHE) [7, 8]. It is known [7] that the xy-component of conductance tensor $\sigma_{xy}$ via Dirac-like spectrum of graphene and consequently additional double degeneration of a zero Landau level (LL) which is common for conduction and valence cones result in a special form of a Hall quantization

$$\sigma_{xy} = \frac{e^2}{2h}, \quad \nu = \pm 2(2k + 1),$$

where $\nu$ is a number of edge modes, $k = 0, 1, 2, ..., \nu$ is a integer number. The inner spin-valley symmetry in graphene leads to four-fold degeneracy of each LL but LL with $k = 0$ has an additional double degeneracy. Nonzero $k$ is given by the expression $k = \left[\frac{\pi n}{\sqrt{2}eB} - \frac{1}{2}\right]$, where symbol $[\cdot]$ stands for the integer part of a number, $n$ is the 2D concentration of electrons in the graphene channel and $eB = \frac{\hbar}{2\pi}$ is the density of the magnetic field flux, threading the 2D surface corresponding to the degree of the k-th LL occupation. In the article [7] have been explained theoretically the peculiarities of IQHE observed experimentally in graphene with p-n junction across the conduction channel.

Hence the FDW induces a p-n junction in graphene at that the wall plane coincides with the p-n junction position [7]. The hallmark [8] of the quantum anomalous Hall (QAH) phase is the existence of chiral edge states, i.e. gapless channels at the edge propagating unidirectionally. The edge spectrum in the QAH phase with Chern number $\mathcal{C} = -1$ confirms existence of one chiral channel per edge [8]. Because [8] the chirality of the edge channel is determined by the sign of Chern number the two QAH phases with $\mathcal{C} = \pm 1$ must have edge channels propagating in opposite directions. Consequently by tuning across the topological phase transition (via Weil Hall state (WHS)) between the two QAH phases one can switch the propagating direction of the edge channel [8]. This can be easily detected with a standard electric transport measurement [8].

Since $\mathcal{P}$ is broken in the 2D WHS state the Chern number $\mathcal{C} = \int_{BZ} \mathcal{II}(k)dk$, where $\mathcal{II}(q) = -23\langle \partial_q u_i \partial_q u_i \rangle$ which is the integral of Berry curvature over the BZ is typically nonzero [8]. Hence because $\mathcal{II}$ is an even function under inversion when the two Weil points are connected by $\mathcal{P}$ the two valley topological charges must be identical and we should have $\mathcal{C} = 1$ or $\mathcal{C} = -1$ [8].

We have investigated the transition from 2D WHS to QAH phases. Because the Weil points in 2D WHS are protected by the mirror $M_y$, breaking $M_y$ will generally remove the Weil points and open the energy gap [8]. This symmetry breaking can be easily achieved by slightly rotating magnetization vector away from the ground-state orientation [8]. In the article [9, 10, 11, 12] we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize ferroics with incommensurately modulated sinusoidal polarization waves $P_3$. The both quantized energies and wave vectors of stationary Schrödinger boundary problem for domain wall are shown to
be related with the Rashba and Dresselhaus quantized energies as well as their wave vectors for electrons in quantized magnetic field. The quantized polarization as well as elastic displacement of stationary Schrödinger boundary problem for domain wall for data storage based on bulk ferroics are found. We have found the quantized solutions of stationarity non-linear high-polynomial Schrödinger or Euler-Lagrange boundary problem for the order parameter as well as elastic displacement in nanosize ferroics. Quantized solutions of one-dimensional Maxwell equations for thin ferroelectric films have been specified which were bonded with stationary solution of cubic and 5-polynomial Schrödinger equation are found. I think in the article [13] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska et.al. were not allowed and consequently were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96).

The quantized polarization as well as energy levels solutions of stationary Schrödinger boundary problem for domain wall for data storage based on interface of polar-active ferroelectric nanofilm with semiconductor are found. The self-consistent stationarity quantized solutions of the both like cubic thus 5-polynomial Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire (LGD) theory theory were presented. We have found the quantized solutions of stationarity non-linear high-polynomial Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor. Quantized solutions of one-dimensional Maxwell equations for thin ferroelectric films have been specified which were bonded with stationary solution of cubic and 5-polynomial Schrödinger equation are found. Recently one can see the increasing attentions regarding complex superstructures with incommensurate nature. The optica of the complex superstructures with incommensurate nature has been intensively explored. A particular examples for those complex superstructures were a modulated structures, a structures of charge density waves or spin density waves, an Abrikosov vortex lattice in the crimping films of the superconducting aluminium in the perpendicular magnetic field in which arise the interference between the vortex period caused the magnetic field and the crimping period [9, 10, 11, 12]. To sum up problem of Quantum Field Theory is the instability of incommensurate phase of Abrikosov vortex lattice of graphene and surface plasmon polaritons of diffraction grating/graphene interface in which arise the interference between the vortex period caused the magnetic field and the period of diffraction metal grating.

In the article [14] the group-theoretical methods and first-principles calculations to analyze a set of coupled structural distortions that underlie the polar charge and orbitally ordered antiferromagnetic ground state of A-site ordered SmBaMn$_2$O$_{6}$ have been developed. The authors show that these distortions play a key role in establishing the ground state and stabilizing a network of domain wall vortices. The polar-active properties of ferroelectric films have been calculated in Landau-Ginsburg-Devonshire theory framework [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 13] with account of polarization vector gradients and surface charges and dipole layers on the interface in the conditions of stability of spontaneous polarization. Under the electric voltage feeds on the electrode or sound of scanning electron microscope the polarization redistributions have been achieved success. Under the increasing the electric voltage the local reversal of polarizations [25, 23, 24, 13] has been achieved which one lied to the creation the intergrown domains in thin films. Under domains recording the velocity of changes of extrinsic electric field $E$ was negligibly small and without loss of accuracy one can suppose $\text{rot} E \approx 0$.

In comparison with the tight-binding method the one-component approximation of LGD theory for polarization and displacement allows the calculation only the lowest optic and lowest acoustic softphonon modes in ferroics. Hence the one-component approximation of LGD theory for polarization and displacement can study quantitatively the soft phonon dispersion in paraelectric SrTiO$_{3}$. However the one-component approximation of LGD theory for polarization and displacement cannot calculate the soft phonon dispersion in multiaxial ferroelectric, pervoskites BaTiO$_{3}$ or (Pb,Zr) TiO$_{3}$ for which the tight-binding method has been applied. When the propagate waves transport high energy then the medium properties can be transformed. The motion equations were non-linear equations for specified of the creation of domain walls for data storage based on polar-active ferroelectric nanofilm with strong optical activity effects.

Under the spontaneous polarization switching in multiferroic as well as ferroelectric the single domain wall as well as the controlled elements of memory of nanodomain structures with increasing density of nanodomains as well as complicate domain recording of information [25, 23, 24, 13].

It is known [26] vibrations with sine-type modulations can be induced by the impulsive stimulated Raman-scattering process resulting in sine-type oscillations. It explains the coherent oscillations observed in transparent compounds under pumping with a photon energy smaller than an optical gap [26]. In an opaque materials the abrupt modification of pump photons can trigger displaced motions of ions towards new coordinates in the excited state resulting in cosine-type oscillations [26].

The problem of the structure stability of GaN is seldom considered in the literature [27]. This is due to the nontypical properties of this compound i.e. short bond length (atomic volume equals 11.4 $\text{Å}$) and relatively high ionicity [27]. In the Philips scale the ionicity of GaN is 0.5 or 0.43 [27] and that makes GaN the most ionic from all the III-V compounds. Almost all discussions of phase transitions of tetrahedrally coordinated compounds consider only the $\beta$-tin and NaCl structures as possible high-pressure phases [27]. The calculations of the total energy of various structures (NaCl, NiAs, $\beta$-Sn, CsCl) compared with wurtzite show that there should be a phase transition to the rocksalt structure around 55
It should be noted that the NiAs phase is very close to the NaCl by stability of domains so both phases are possible [27]. A phase transition possibly to a NaCl structure has been found around 48 GPa at increasing pressure and between 30 and 20 GPa at the downstroke [27].

The growth conditions of GaN were selected for a stable growth on substrates oriented towards the c-direction [28]. Numerous hills in the form of cones and bubbles were clearly visible on the surface [28]. There were areas with different shades of blue and grey and even had a yellow area with respect of substrates oriented of growth [28].

The physical properties of wide bandgap group-III quantum well systems are under investigation due to their application to light emitters and semiconductor lasers in the ultraviolet, blue, and green wavelength regions. Ultraviolet light-emitting diodes and lasers have recently obtained considerations due to applications to the compact biological detection systems, analytical devices, and medical diagnostics. A number of light-emitting diodes and laser diodes have been demonstrated [29, 30]. However, these structures are in the developmental stage, and there are many questions with respect to the performance and device configurations.

Realizing the deep-ultraviolet semiconductor-based light-emitting diodes provides light sources for various applications, for instance to the biological detection and the data storage. Although such devices basically need a AlGa1−xN-based quantum well with high Al contents, their fundamental optical properties remain under discussion [31, 32, 12, 33, 16, 34, 35, 36, 37, 38, 39, 10, 11, 40]. It has been proved experimentally that the surface emission from [0001]-oriented AlGa1−xN is quite weak because of the predominant optical polarization along the [0001] c direction [41, 42, 43]. The explanation of these effects may be found from the difference of structures of the valence bands in AlN and in GaN. In wurtzite GaN or AlN, the degeneracy of the p-like states at the Γ point is lifted by both crystal-field splitting and spin-orbit splitting leading to forming three valence bands at the Brillouin zone center.

Since AlN has a negative crystal field splitting energy, while GaN has a positive one, these splittings lead to the ordering of the valence band in AlN: Γ7, ΓΓ9, and Γ97. Whereas we have Γ97, Γ7, and Γ79 in GaN [44, 45]. Therefore, the topmost of the valence band in AlN has the crystal field split off holes with p-like states, while the topmost in GaN has the heavy holes with p-like and p-like states, where the axis z is directed along the hexagonal axis. Therefore, the emission from AlGa1−xN with high (low) Al-content is polarized along (perpendicular to) the c axis.

Recently, many studies have been focused on the potential application of nanostructures, such as photonic crystal structures, nanoholes, nanodots, and nanorods [46, 47, 48, 49, 50, 51, 26, 52, 8, 53, 54, 55, 56, 3, 57, 58, 59, 18, 60, 61, 62, 37, 63]. In the studies of the technology involving the photonic band gap, it seems that, in the case of dielectric rod nanoarrays or nanocolumns [39, 10, 64, 65, 66, 67, 68, 38], a large gap is opened for the TM mode, but not for the TE one [45, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 17]. Thus, with this type of structures for laser applications, the light source in the TM mode is obtained.

In the c-plane of InGaN/GaN quantum well systems, the compressive strain is induced in the active layer, and the light is TE-polarized [64, 85, 86, 87]. Furthermore, there is a strong internal electric field caused by the spontaneous and piezoelectric polarization charges at the interfaces of the c-plane of the InGaN/GaN quantum well. This phenomenon leads to the quantum confined Stark effect, decreases the internal quantum efficiency, and leads to the emission spectrum which is red-shifted [88, 40, 11, 89, 90, 91, 92].

In some studies [93, 66, 15, 65, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103] of interface polarization charges, alloy materials were used to make a better performance. Many works have focused on the nonpolar and semipolar planes [104, 105, 15, 87, 86]. These results have testified that the light emission will be polarized, and the quantum confined Stark effect will be reduced. However, due to a higher cost of the a- and m-plane substrates, it would be better to use the c-plane substrate. In work [97, 98, 106, 107], the c-plane of the InGaN/AlGaN quantum well structure was considered instead of that of InGaN/GaN in order to obtain a tensile strain in the quantum well layer. The previous studies and calculations have shown that the [Z]-like state is generated in nitride materials, if the quantum well layer is under a tensile biaxial strain.

Besides the nitride-based devices, the group-II oxides have been considered for highly efficient laser diodes [106, 107] and high-performance field-effect transistors [108, 109]. The induced piezoelectric field plays a significant role for both band structure and optical gain [110, 111, 112]. However, the orientation of a crystal structure significantly modifies the band structure through the strain effect [113]. It has been proved experimentally that the growth along crystal directions different from the [0001] direction leads to an increase in the quantum efficiency by decreasing the strain-induced electric field in the quantum well region, possibly leading to the ways of obtaining highly efficient white laser diodes [114]. There are the theoretical works studying the effects of crystal orientation on the piezoelectric field in a strained wurtzite quantum well [112]. However, the piezoelectric effect consists not only of a strain-induced polarization; it also takes the response of both electric field and polarization on the strain into consideration. These effects were studied in paper [112].

A deeper understanding of the influence of band structures on optical properties should help one to answer many questions. In addition, the interesting effects of strong electron-hole Coulomb interaction are presented in these materials. Many-body interactions lead to effects, which consist the screening, dephasing, bandgap renormalization, and phase-space filling [115, 116, 117, 118].

A general phenomenon of Coulomb enhancement may be explained as follows. Due to the Coulomb attraction, an electron and a hole have a larger tendency to be located near each other, than in the case of noninteracting
particles. This increase of the interaction duration leads to an increase of the optical transition probability.

In the article the presented phenomena arising from the interplay between the polarity and electronic structure include two dimensional electron gases at the interface of band (LaAlO$_3$/SrTiO$_3$) or band or Mott insulators and polarization controlled electron tunneling across ferroelectric-semiconductor or half-metal interfaces (PtBiO$_3$/La,SrMnO$_3$, BaTiO$_3$/SrRuO$_3$) [18, 19, 24, 23, 119]. The analytical calculations of polar active properties (including local polarization reversal) in the proper and incipient ferroelectric-dielectric thin films within Landau-Ginsburg-Devonshire (LGD) phenomenological theory with consideration of polarization gradient and intrinsic surface energy, interface dipoles, and free charges were presented in the works [18, 19, 24, 23, 119]. Hence the latter works provides a framework to link the mesoscopic LGD semiconductor theory to the first principle calculations that can reveal the detailed structure of the electrostatic field structure at the interface.

The surface energy is assumed to be a positively defined quadratic form

$$L_{G D} \text{ free-energy density is a sum of Landau, gradient, and surface energies}$$

is due to the Au atoms in the hollow position that get closer to graphene and do not break the sublattice symmetry [18, 19, 24, 23, 119]. The gradient energy \(\sigma\) is the average crystal structure has inversion symmetry [46]. In order to explain this phenomenological space dependent dielectric and gyration tensors being invariant with respect to the superspace group of Rb$_2$ZnBr$_4$, Rb$_2$ZnCl$_4$, K$_2$SeO$_4$, (NH$_4$)$_2$BeF$_4$, (N(CH$_3$)$_4$)$_2$MnCl$_4$ crystals has been measured along three directions in a temperature interval from 400K to 50K [46]. In the incommensurate phase already a nonvanishing element of the gyration tensor has been observed despite the fact that the the average crystal structure has inversion symmetry [46].

It is known that the spin-orbit coupling originates from gradients of the Coulomb potentials in the atomic cores [132, 133, 134, 135]. The spin-orbit coupling have been presented by \(\Delta = \frac{\hbar}{2 m V} (\psi_\alpha | \nabla \mathbf{V} \times \mathbf{p} | \psi_\beta)\), where \(V\) is the microscopic crystal potential of graphene. In graphene we would have \(\Delta = 0\) if the basis functions \(|\psi_\alpha\rangle\) and \(|\psi_\beta\rangle\) were made up of basis of pure \(\pi\) or \((p_x, p_y)\) orbitals. However spin-orbit coupling induces a mixing of the \(\pi\) or \((p_x, p_y)\) orbitals [136] in graphene that contributes to \(\Delta\) in second order of spin-orbit coupling. In the our case [62, 16, 37, 63, 137, 64, 65, 48, 66, 67, 138, 68] the strain mediates a coupling between intrinsic-spin and orbital dynamics and the lowest order contribution \(p^{SS}_{2n}(\epsilon_{xx} + \epsilon_{yy})\delta_{i} = \sigma_{2}\) constitutes a renormalization of the intrinsic spin-orbit coupling \(p^{SS}_{2n}\). In the article \([47, 52, 139, 8, 53, 140, 141, 105, 104, 55, 56, 3, 57]\) a creation of giant spin-orbit splitting (\(~ 100\) meV) of the graphene Dirac cone up to the Fermi energy are shown to be related with Au intercalation at the graphene-Ni interface. Photoelectron spectroscopy \([142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 5, 153, 154, 155]\) reveals the hybridization with Au 5d states as the source for this giant splitting. A sharp graphene-Au interface at the equilibrium distance accounts for only \(~ 10\) meV spin-orbit splitting and enhancement is due to the Au atoms in the hollow position that get closer to graphene and do not break the sublattice symmetry\([114, 113, 112, 156, 157, 158, 159, 160, 161, 111, 2, 6, 162, 110, 109, 108, 44, 143, 117, 116, 115]\).

### 3 Building a free-energy functional from atomically resolved imaging: Atomic-scale phenomena in Os-doped RaFeO$_3$ with magnetic field

We have described the four-sublattice model (FSM) in accordance with experimental results [1]. The conventional LGD free-energy density is a sum of Landau, gradient, and surface energies [1]:

$$G = \int_V (G_{\text{Landau}} + G_{\text{grad}}) + G_{S}. \quad \text{(1)}$$

Landau free-energy expansion containing the quadratic and bilinear contributions of the A-cations displacements \(A_i\) (\(i = 1 \sim 4\)) in ABO$_3$ perovskite with \(m3m\) parent phase is [1]

$$G_{\text{Landau}} = \frac{\alpha}{2} A_i^2 + \mu(A_1 A_2 + A_2 A_3 + A_3 A_4 + A_4 A_1) + \eta(A_1 A_3 + A_2 A_4) + \frac{\beta}{4} A_i^2 A_j^2 + \frac{\gamma}{2} (A_1^2 A_2^2 + A_2^2 A_3^2 + A_3^2 A_4^2 + A_4^2 A_1^2) + \frac{\delta}{2} (A_1^2 A_3^2 + A_2^2 A_4^2 + A_3^2 A_1^2 + A_4^2 A_2^2) + \lambda A_1 A_2 A_3 A_4 + \ldots \quad \text{(2)}$$

Here we assume that only the first term in Eq. (2) has a temperature-dependent coefficient \(\alpha = \alpha_T(T - T_c)\) and all constants can depend on the global or local content of impurity (e.g. Os atoms) [1].

The gradient energy \(G_{\text{grad}} = g_{ijkl} \frac{\partial A_i}{\partial x_j} \frac{\partial A_j}{\partial x_l}\) will be considered in the simplest isotropic approximation for the gradient coefficient tensor \(g_{ijkl}\) and that all physical quantities are \(x\) dependent [1]:

$$G_{\text{grad}} = g_{ijkl} \frac{\partial A_i}{\partial x_j} \frac{\partial A_j}{\partial x_l} \approx g (\frac{\partial A_i}{\partial x_j})^2. \quad \text{(3)}$$

The surface energy is assumed to be a positively defined quadratic form [1]

$$G_S = \alpha_S (A_1^2 + A_2^2 + A_3^2 + A_4^2). \quad \text{(4)}$$

Under research of Dzyaloshinsky substitution [1]
Hall conductivity in the experiment (Ref. [2020: Pages 1-32: orcid.org 51x180]B where the order parameters
the sublattice asymmetry constant 
we have found the following the Euler-Lagrange equations
analytically the Euler-Lagrange equations with consideration of the Landau levels by variation of the energy (8). Hence
stability conditions are
the order-parameter field
range order parameter and some predefined relationship between experimentally determined atomic coordinates and
Hence the analysis often relies on the postulated form of macroscopic Landau-Ginsburg energy for the ferroic long-
and making elementary algebraic transformation listed in Ref. [1] one could rewrite Eq. (2) as follows:
\[
G_{\text{Landau}} = \frac{\alpha^2}{2}B_1^2 + \frac{\mu^2}{2}(B_2^2 + B_3^2) + \frac{\gamma^2}{2}B_2^2 + \frac{\lambda^2}{2}(B_2^2 B_3^2 + B_2^2 B_4^2) + \eta \eta^* B_1 B_2 B_3 B_4.
\]
(6)
The expansion coefficients are [1]
\[
\alpha^* = \alpha + 2\mu + \eta, \quad \mu^* = \alpha - \eta, \quad \eta^* = \alpha - 2\mu + \eta, \quad \beta^* = \beta^2 + \frac{\delta}{4} + \frac{\gamma}{4} + \frac{\lambda}{4},
\]
\[
\gamma^* = \frac{\beta^2}{4} + \frac{\delta}{4} + \frac{\gamma}{4} + \frac{\lambda}{4}, \quad \lambda^* = \frac{\beta^2}{4} - \gamma - \frac{\delta}{2} - \frac{\lambda}{2}.
\]
(7)
Hence the analysis often relies on the postulated form of macroscopic Landau-Ginsburg energy for the ferroic long-
range order parameter and some predefined relationship between experimentally determined atomic coordinates and
the order-parameter field [1]. Hence the R and O phases coexistence (that is observed by STEM) can be realized in the
case of their energies equality. The coexistence condition \(G_R = G_O\) gives \(\alpha - \eta = \alpha + 2\mu + \eta\) i.e. \(-\eta = \mu\) and the phase
stability conditions are \(\alpha + 2\mu + \eta < 0, \quad \alpha - \eta < 0\) and \(\beta + 2\gamma + \delta + \lambda > 0\) [1]. In the case of the weak deviations from
the phase equilibrium \(-\eta = \mu\), i.e. when the condition \(\eta + \mu + \varsigma = 0\) takes place along with the inequality \(|\varsigma| < \mu\),
one could write the free energy (6) in the following dimensionless form [1]:
\[
G_{12} \approx -(\alpha + \mu)B_3^2 \int \left[ -(1-c) \frac{\delta^2}{2} (1-c) \frac{\delta^2}{2} + \frac{b_1^2 + b_2^2}{4} + \frac{b_1^2 b_2^2}{2} + \frac{\delta \eta}{2} \right] (\frac{\delta}{\delta x})^2 + \frac{(\delta b_2)}{2} (\frac{\delta b_2}{\delta x})^2 + \frac{\beta^*}{2} \mu_0 \mu_0 \mu_0 \sigma^2
\]
where the order parameters \(B_i = B_S b_i\) (\(i = 1, 2\)), the spontaneous value \(B_S = 2\sqrt{\frac{\alpha + 2\mu + \eta}{\beta + 2\gamma + \delta + \lambda}}\), dimensionless coupling
constant \(\chi = (\frac{\beta^*}{4} + \frac{\delta}{4} + \frac{\gamma}{4} + \frac{\lambda}{4})^{-1}\), the gradient coefficient \(h = B_2^2 g\), and the parameter \(c \equiv \frac{\delta}{\alpha + \mu}\) is
the sublattice asymmetry constant [1]. To study the conditions between coexisting R and O phases we have solved
analytically the Euler-Lagrange equations with consideration of the Landau levels by variation of the energy (8). Hence
we have found the following the Euler-Lagrange equations
\[
-(1-c) + b_1^2 + \chi b_1^2 (b_1) = 0, \quad -(1-c) + b_2^2 + \chi b_2^2 (b_2) = 0,
\]
from which we have specified the \(b_{2i}\) variable via \(b_{2i}\) variable
\[
b_{2i} = (1-c) - h b_i^2 - \chi b_i^2.
\]
(10)
Hence we have found the following equation

\[(1 - \chi^2) b_{1s}^2 \Psi + (1 - \chi) h c k_s^2 \Psi = (1 - c) \Psi,\]  
(11)

where \(\Psi \equiv (b_{1s}, b_{2s}).\) The covariant derivative \(\nabla = \partial + (i e/\hbar c)A\) includes the vector potential in the symmetric gauge \(A^\text{ext} = (-\mathcal{H}_x, \mathcal{H}_y, \mathcal{H}_z)\) corresponding to the external magnetic field applied perpendicular to the plane along the positive \(z\) axis.

\[(1 - \chi^2) b_{1s}^2 \Psi + (1 - \chi) h c k_s^2 \Psi = (1 - c) \Psi,\]  
(12)

in which introducing \(y^* = y + y_0\) where \(y_0 = s^2 k_s, s^2 = \hbar c / \mathcal{H}_x\) we have found

\[(1 - \chi^2) b_{1s}^2 \Psi + (1 - \chi) h s^{-2}(s^2 - 2s^2 y^*) \Psi = (1 - c) \Psi,\]  
(13)

as well as

\[(1 - \chi^2) b_{1s}^2 \Psi + (1 - \chi) h s^{-2}(\xi^2 - \xi^2 / \hbar c) \Psi = (1 - c) \Psi,\]  
(14)

where \(\xi = s^{-1} y^*.\) We seek the solution of Eq. (15) in the form \(\Psi = \sum_n C_n \exp \int^{\xi/2} H_s(\xi)\)

\[= \sum_n \int_{-\infty}^{\xi} d\xi \exp \int^{\xi/2} H_s(\xi)(1 - \chi) h s^{-2}(\xi^2 - \xi^2 / \hbar c) C_n \exp \int^{\xi/2} H_s(\xi),\]  
(16)

with taking into account the condition of normalization of wave function and the following recurrence relations

\[\int_{-\infty}^{\xi} d\xi \exp \int^{\xi/2} H_s(\xi) \exp \int^{\xi/2} H_s(\xi) = \begin{cases} 0 & n \neq m, \xi \neq 1 \chi \Psi_s(\xi) = \sqrt{\frac{\xi}{\hbar c}} \Psi_{n-1} + \sqrt{\frac{\xi+1}{\hbar c}} \Psi_{n+1}, \xi, \frac{d}{d\xi} \Psi_s(\xi) = \sqrt{\frac{\xi}{\hbar c}} \Psi_{n-1} - \sqrt{\frac{\xi+1}{\hbar c}} \Psi_{n+1}, \end{cases}\]

we have specified the following algebraic equation

\[(1 - \chi^2) b_{1s}^2 + 2(1 - \chi) h s^{-2}(n + \frac{1}{2}) = (1 - c),\]  
(17)

from which we have found

\[b_{1s} = \pm \sqrt{\frac{(1 - c)}{(1 - \chi^2)}} - \frac{2h s^{-2}}{(1 + \chi)}(n + \frac{1}{2}),\]  
(18)

as well as

\[b_{2s} = \pm \sqrt{(1 - \chi) - 2h s^{-2}(n + \frac{1}{2}) - \frac{2h s^{-2}}{(1 + \chi)}(m + \frac{1}{2})},\]  
(19)

where \(n, m = 0, 1, 2, 3, 4, \ldots.\) The solution (19) can be rewritten also like

\[b_{2s} = \pm \sqrt{(1 - \chi) - 2h s^{-2}(m - n) - \frac{(1 - \chi)}{(1 + \chi)}(m + \frac{1}{2})},\]  
(20)

where \(s^{-2} = \frac{e \mathcal{H}_s}{\hbar c}.\) The solution (18) can be found also like

\[b_{1s} = \pm \sqrt{\Delta_{1s}^2 + 2nh e \mathcal{H}_s^2 / (h c(1 + \chi))},\]  
(21)

where \(\Delta_{1s}^2 = \frac{(1 - c)}{(1 - \chi^2)} + e \mathcal{H}_s^2 / (h c(1 + \chi)).\) Hence the Hall conductivity can be found like for graphene in the form of expression [165, 166, 167]

\[\sigma_{xy} = -\frac{e^2 N_s \text{sgn}(e \mathcal{H}_s)}{2 \hbar n} [1 + 2 \sum_{\mu = -n}^{\infty} \theta(\mu - M_n)] = -\frac{e^2 N_s \text{sgn}(e \mathcal{H}_s)}{n} (1 + 2[\frac{\mu^2 c}{2h |e \mathcal{H}_s|}]),\]  
(22)

via which we have specified the Integer Hall conductivity for thin Os_{x}Ra_{1-x}FeO_{3} nanofilms in the form

\[\sigma_{xy}^{1s} = \frac{-e^2 N_s \text{sgn}(e \mathcal{H}_s)}{2 \hbar n} (1 + 2[\frac{\mu c(1 + \chi)}{2h |e \mathcal{H}_s|}]).\]  
(23)

The solution (20) can be found also like
Let us show how the generation of the gap affects the form of the Hall conductivity with the digamma function \( \Psi(\Delta_x^2) \). For which we have specified the Integer Hall conductivity for thin Os, Ra1, FeO, nanofilms in the form

\[
\sigma_{xy}^{\gamma} = \frac{\gamma^2 N_c \sigma_x}{2 \pi n} \left( 1 + 2 \left[ \frac{\pi^2 \hbar c (1 + \chi)}{2 n e |\mathcal{E}|} \right] \right),
\]

where \([x]\) denotes the integer part of \( x \), \( N_f = 2 \), so \( \nu_{\mathcal{E}} = 2 n + 1 \).

4 Phase transition and quantum Hall effect in graphene

4.1 Unconventional Integer Quantum Hall Effect in Graphene

The low-energy quasiparticles excitations in graphene are conventional described in terms of a four-component Dirac spinor \([165, 166, 167]\) \( \Psi^\dagger = (\psi^K, \psi^{K_1}, \psi^{K_2}, \psi^{K_3}) \) which combines the Bloch states with spin \( \sigma = \pm 1 \) on the two different sublattices \((A, B)\) of the trigonal graphene band structure and with momenta near the two inequivalent \((K, K')\) points \((K, K')\) at the opposite corners of the two-dimensional Brillouin zone. The free quasiparticle Hamiltonian can be recast in the relativisticlike form with the Fermi velocity \( v_F = 10^6 m/s \) like velocity of light in relativistic physics \([165, 166, 167]\),

\[
H_0 = -v_F \int d^2 \mathbf{r} \Psi^\dagger \gamma^0 \hbar \nabla_x + \gamma^3 \hbar \nabla_y \Psi, \tag{26}
\]

where \( \Psi^\dagger = \Psi^\dagger \gamma^0 \) is the Dirac conjugated spinor and summation over spin \( \sigma \) is understood. In Eq. (26) \( \gamma^0 \) with \( \nu = 0, 1, 2 \) are \( 4 \times 4 \) gamma matrices belonging to a reducible representation \( \gamma^0 = \tilde{\tau}_3 \otimes (\tau_3, \epsilon \tau_2, -\epsilon \tau_1) \) where the Pauli matrices \( \tilde{\tau}, \epsilon \) act in the subspaces of the valley \((K, K')\) and sublattices \((A, B)\) indices, respectively. The matrices satisfy the usual anticommutation relations \( \{\gamma^0, \gamma^0\} = 2 \delta_{\mu \nu}, \gamma^0 = 1, -1, 1 \), \( \mu, \nu = 0, 1, 2 \). The covariant derivative \( \nabla = \partial + (\epsilon \hbar/c) \mathbf{A} \) includes the vector potential in the symmetric gauge \( A^{\sigma \nu} = \left(-By/2, Bx/2\right) \) corresponding to the external magnetic field applied perpendicular to the plane along the positive \( z \) axis. In the four-component spinor representation the Coulomb interaction has the form \([165, 166, 167]\),

\[
H_{\text{int}} = \frac{b_{\gamma}}{2} \int d^2 \mathbf{r} d^2 \mathbf{r}' \Psi^\dagger (\mathbf{r}) \gamma^0 \Psi (\mathbf{r}) \frac{\mathbf{r} - \mathbf{r}'}{\left| \mathbf{r} - \mathbf{r}' \right|^2} \Psi^\dagger (\mathbf{r}) \gamma^0 \Psi (\mathbf{r}),
\]

where the coupling \( g = e^2/\epsilon_0 \hbar v_F \) and \( \epsilon_0 \) is the dielectric constant (our convention is \( \epsilon > 0 \)). The total Hamiltonian \( H_{\text{tot}} = H_0 + H_{\text{int}} \) possesses \( U(4) \) symmetry discussed in the articles \([165, 166, 167]\). The chemical potential is introduced through adding the term \(-\mu \gamma^0 \Psi = -\mu \Psi^\dagger \Psi \) in \( H_{\text{tot}} \) (this term preserves the \( U(4) \) symmetry). The Zeeman interaction term is included by adding the term \( \mu_B B \bar{\Psi} \gamma_0 \sigma_3 \Psi = \Psi^\dagger \sigma_3 \Psi \), where \( \sigma_3 \) matrix acts on spin indices. Here \( \mu_B = e\hbar/2mc \) is the Bohr magneton and we took into account that the Lande factor for graphene \( g_t \approx 2 \).

Let us first consider case with no spin splitting (the Zeeman term is ignored). We have derived the thermodynamic potential per unit area in a strong magnetic field \( B \), when the dynamics of the lowest Landau level (LLL) dominate \([165, 166, 167]\),

\[
\Omega' (\Delta, \mu) = \frac{1}{2\pi} \left[ \Delta f (\Delta, \mu) - \frac{bl(\mu B)}{2} \frac{f^2 (\Delta, \mu) + 2 f' (\Delta, \mu)}{\Delta} \right] \times \Delta \left( \frac{\gamma^{\nu} g^{\nu + (\mu + \Delta)}}{2nT} + \frac{1}{2} \right) + \ln \Gamma \left( \frac{\gamma^{\nu} g^{\nu + (\mu + \Delta)}}{2nT} + \frac{1}{2} \right),
\]

where \( \mu_B = e\hbar/c \) is the magnetic length, \( L(B) = \sqrt{\hbar v_F^2 e B}/c \) is the Landau scale, \( \Gamma(x) \) is the Euler gamma function, \( \gamma \) is a LLL impurity scattering rate, and the function \( f (\Delta, \mu) \) is \([165, 166, 167]\),

\[
f (\Delta, \mu) = \frac{1}{\pi} \int_0^\infty dx \Psi \left( \frac{\gamma^{\nu} g^{\nu + (\mu + \Delta)}}{2nT} + \frac{1}{2} \right) - \Delta, \Delta \rightarrow -\Delta \right]\]

with the digamma function \( \Psi(x) = \frac{d}{dx} \ln \Gamma(x) \). The dimensionless parameter \( b \) in Eq. (28) reads

\[
b = \frac{g}{2\pi} \int_0^\infty \frac{dk \exp(-k)}{1 + k \chi_0},
\]

where \( \chi_0 \approx 0.56 \sqrt{2\pi} g \). The gap equation \( \partial \Omega' / \partial \Delta = 0 \) for \( \Delta \) takes the form \([165, 166, 167]\),

\[
\Delta - bL(B)f (\Delta, \mu) = 0.
\]

Let us show how the generation of the gap affects the form of the Hall conductivity \( \sigma_{xy}^{\gamma} \), valid for large \( B \). For large enough \( B \) (or small enough \( \gamma \) scattering rate) it becomes a strong first order phase transition \([165, 166, 167]\),

\[
\sigma_{xy}^{\gamma} = -\frac{2e^2 g \mu_B}{\pi \hbar} \int d^2 \mathbf{r} \Psi \left( \frac{\gamma^{\nu} g^{\nu + (\mu + \Delta)}}{2nT} + \frac{1}{2} \right) - \gamma^{\nu} \frac{\mu_B}{2nT} \Psi \left( \frac{\gamma^{\nu} g^{\nu + (\mu + \Delta)}}{2nT} + \frac{1}{2} \right) + \Delta \rightarrow -\Delta \right],
\]
where $\gamma_{tr}$ is the transport scattering rate. To illustrate the role of opening the gap let us consider the clean limit case $\gamma = \gamma_{tr} = 0$ at zero temperature, when Eq. (32) takes the form [165, 166, 167]

$$\sigma_{xy}^* = -\frac{2e^2}{h} sgn(\mu) sgn(eB) \delta [\mu - \Delta(\mu, B)].$$

(33)

The lowering the degeneracy of the LLL with generating the gap in graphene is connected with the spontaneous breakdown of the initial $U(4)$ symmetry down to $U(2)_a \times U(2)_b$. The expression for diagonal conductivity is found in the articles [165, 166, 167]

$$\sigma_{xx}^* = -\frac{2e^2}{h^2} \frac{\gamma^2}{\gamma^2 + \Delta^2},$$

(34)

for the trigonal band structure of graphene $E = \pm \sqrt{2hv_F^2|\varepsilon| + \Delta^2}$. Hence the thermodynamic potential $\Omega$, the Hall conductivity $\sigma_{xy}$ and the diagonal conductivity $\sigma_{xx}$ are now [165, 166, 167]

$$\Omega(\Delta, \mu) = \frac{1}{2} [\Omega'(\Delta, \mu_+) + \Omega'(\Delta, \mu_-)],$$

(35)

$$\sigma_{xy} = \frac{1}{2} [\sigma_{xy}(\mu_+) + \sigma_{xy}^*(\mu_-)],$$

(36)

$$\sigma_{xx} = \frac{1}{2} [\sigma_{xx}(\mu_+) + \sigma_{xx}^*(\mu_-)],$$

(37)

where $\mu_{\pm} = \mu \pm \delta_\mu$ with $\delta_\mu = \mu_B B$ being the Zeeman term. This term breaks explicitly the $U(4)$ symmetry down to $U(2)_a \times U(2)_b$ at phase transitions driven by magnetic field. The latter is obtained by a simple compilation of the experimentally observed transition points between the corresponding plateaus in Hall conductivity. The best fit that was found is [165, 166, 167]

$$\mu = 0.5(\nu_B - \nu_0) + 7.0 sgn(\nu_B - \nu_0) \sqrt{|\nu_B - \nu_0|}.$$  

(38)

### 4.2 Unusual quantization of the Hall conductivity in graphene

Hence the expression for Hall conductivity we have found like

$$\sigma_{xy} = -\frac{e^2 N_{s,g}(eB)}{4\pi} [\tanh \frac{\mu + \Delta}{2T} + \tanh \frac{\mu - \Delta}{2T} + 2 \sum_{n=1}^{\infty} (\tanh \frac{\mu + M_n}{2T} + \tanh \frac{\mu - M_n}{2T})],$$

(39)

where $\tanh \frac{\omega - \mu}{2T} = 1 - 2n_F(\omega)$ so we have rewritten Eq. (39) via Fermi distribution

$$\sigma_{xy} = -\frac{e^2 N_{s,g}(eB)}{2\pi} \sum_{n=0}^{\infty} (2n + 1) [n_F(M_n) + n_F(-M_n) - n_F(M_{n+1}) - n_F(-M_{n+1})],$$

(40)

where $M_n = \pm \sqrt{2nhv_F^2|\varepsilon| + \Delta^2}$. Now we rewrite Eq. (39) as follows

$$\sigma_{xy} = -\frac{e^2 N_{s,g}(eB) sgn \mu}{2\pi h} \nu_B,$$

(41)

with the filling factor

$$\nu_B = \frac{\mu}{2} [\tanh \frac{\mu + \Delta}{2T} + \tanh \frac{\mu - \Delta}{2T} + 2 \sum_{n=1}^{\infty} (\tanh \frac{\mu + M_n}{2T} + \tanh \frac{\mu - M_n}{2T})],$$

(42)

where $h = 2\pi \hbar$, and using that tanh$(\omega/2T) = sgn(\omega)$ at $T \rightarrow 0$ we obtain from Eq. (44)

$$\sigma_{xy} = -\frac{e^2 N_{s,g}(eB)}{2\pi h} [1 + 2 \sum_{n=1}^{\infty} \theta(\mu - M_n)] = -\frac{e^2 N_{s,g}(eB)}{h} (1 + 2 [\frac{\mu c}{2h|eB|v_F}]),$$

(43)

where $[x]$ denotes the integer part of $x$, $N_f = 2$, $v_B = 2n + 1$.

### 4.3 Calculation of the conductivity

In the bare bubble approximation the expression for the diagonal conductivity in the limit of $B \rightarrow \infty$ can be obtained from Eqs. (3.11), (3.12) in the second paper in Ref. [166]

$$\sigma_{xx}^* = \frac{2e^2}{\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{4T \cosh \frac{\omega}{2T}} \left[ \frac{\gamma_F}{\gamma_{tr}^2 + (\omega - \Delta)^2} + \frac{\gamma_{tr}}{\gamma_{tr}^2 + (\omega + \Delta)^2} \right],$$

(44)

the integrals in this expression can be evaluated exactly as follows

$$I = \int_{-\infty}^{+\infty} \frac{d\omega}{\cosh \frac{\omega}{2T}} \frac{\gamma_F}{\gamma_{tr}^2 + (\omega - \Delta)^2} = i \Re e \int_{-\infty}^{+\infty} dt \exp^{-\gamma_{tr} t} \int_{-\infty}^{+\infty} \frac{d\omega}{\cosh \frac{\omega}{2T}} \exp^{i[\omega - (\mu - \Delta)]},$$

(45)
The integral over $x$ is evaluated by means of the formula from Ref. [165]. Then we get

$$I = 4\pi T^2 \int_0^\infty \frac{d\omega}{\sinh(\pi T \omega)} \frac{e^{i\omega x} [\Psi_{\mu}] e^{\gamma \omega}}{\omega}.$$  \hspace{1cm} (46)

This integral can be evaluated by differentiating from Ref. [165] which yields

$$I = 4\pi T^2 \int_0^\infty \frac{d\omega}{\sinh(\pi T \omega)} \frac{e^{i\omega x} [\Psi_{\mu}] e^{\gamma \omega}}{\omega} = \frac{2}{\pi} \Re e \Psi(\frac{[\gamma \omega x + (\mu - \Delta)]}{2\pi T} + \frac{1}{2}).$$  \hspace{1cm} (47)

Thus we obtain

$$\sigma^*_{xx} = e^{i\omega x} \frac{2}{\pi} \Re e \Psi(\frac{[\gamma + x + (\mu - \Delta)]}{2\pi T} + \frac{1}{2}) + \Psi(\frac{[\gamma x - \mu - \Delta)]}{2\pi T} + \frac{1}{2}).$$  \hspace{1cm} (48)

One can check that for $\mu = 0$ and $T \rightarrow 0$ Eq. (48) reduces to Eq. (34). Now we derive the expression for the dc Hall conductivity. In the limit $B \rightarrow \infty$, Eqs. (3.14), (3.15) in the second paper in Ref. [166] yield

$$
\sigma_{xy}^* = e^{i\omega x} \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\gamma x + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} + \frac{\gamma x - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right].
$$  \hspace{1cm} (49)

We first consider the terms with arctan functions taking derivative with respect to $\Delta$:

$$
\frac{\partial \sigma_{xy}^*}{\partial \Delta} = -e^{i\omega x} \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\gamma x + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} - \frac{\gamma x - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right].
$$  \hspace{1cm} (50)

Then we use Eq. (47) and find

$$
\frac{\partial \sigma_{xy}^*}{\partial \Delta} = -e^{i\omega x} \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\gamma x + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} - \frac{\gamma x - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right].
$$  \hspace{1cm} (51)

Therefore

$$
\sigma_{xy}^{(1)} = -e^{i\omega x} \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\gamma x + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} - \frac{\gamma x - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right].
$$  \hspace{1cm} (52)

Here we took into account the fact that because of the condition $\sigma_{xy}^*(\Delta = \infty) = 0$ (see Eq. (49)) and the known asymptotics of the $\Psi$ function the integration constant equals zero. As to the integrals of the two first terms in square brackets in Eq. (49) they are calculated by means of the formula

$$\int_0^\infty \frac{d\omega}{\cosh(\omega / T)} = -\frac{2}{\pi} \Re e \Psi(\frac{[\gamma r^+ + (\omega - \Delta)]}{2\pi T} + \frac{1}{2}).$$  \hspace{1cm} (53)

Its derivation is as follows:

$$\int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\gamma x + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} - \frac{\gamma x - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right] = \Re e \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{1}{\omega - \gamma r^+ + (\omega - \Delta)^2} - \frac{1}{\omega - \gamma r^- + (\omega - \Delta)^2} \right] = 2T \Re e \int_0^\infty \frac{d\omega}{\cosh(\omega / T)} \left[ \frac{\omega + (\mu - \Delta)}{\gamma r^+ + (\omega - \Delta)^2} - \frac{\omega - (\mu - \Delta)}{\gamma r^- + (\omega - \Delta)^2} \right] = 4\pi T^2 \int_0^\infty \frac{d\omega}{\sinh(\pi T \omega)} \frac{(\omega - \Delta)^2}{\gamma r^+ + (\omega - \Delta)^2} = 4\pi T^2 \int_0^\infty \frac{d\omega}{\sinh(\pi T \omega)} \frac{(\omega - \Delta)^2}{\gamma r^- + (\omega - \Delta)^2}.$$

Combining this contribution with that in Eq. (52) we arrive at Eq. (32).

The free energy expansion in powers of order parameters is employed in Landau-Ginsburg-Devonshire (LGD) free energy and can be used in the phase-field modeling of macro and nanosized ferroelectrics [1]. Understanding of ferroic behavior at surfaces, interfaces, and defects as well as the nature of ferroelectric states considerably advanced in the last decades with the advancement of scanning transmission electron microscopy (STEM) [1]. Probing the unit-cell level symmetry breaking via STEM allowed the determination of direct atomic positions, from which the spatial distributions of order parameters fields can be mapped [1]. Hence here we derive a model LGD-type free energy describing directly observable degrees of freedom available from atomic-resolution STEM [1]. We have proposed a theoretical four-sublattice model (FSM) for analytical description of cation displacement in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type, that explain the coexistence of rhombohedral (R), orthorhombic (O), and spatially modulated (SM) phases observed by atomic-resolution STEM [1]. Using this approach we atomically resolve and theoretically model the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in perovskites of Os,Ra$_{1-x}$FeO$_3$ polymorphs with magnetic field. Hence for over 50 years of research of ferroic materials the properties of these materials were explored using the combination of scattering techniques that provides the information on the nature and symmetry of order parameters and macroscopic measurements that provided the information on the corresponding expansion coefficients and the nature of phase transitions [1].
Figure 2. (Color online) A colored Hofstadter butterfly (Ref. [2]): This figure represents the phase diagram of Bloch electrons in a uniform magnetic field. The horizontal axis indicates the chemical potential and the vertical axis the flux through the system. Each color corresponds to a distinct structural phase transition with a particular quantized value of the Hall conductance.

bands - the (first) Chern number. In this model, time-reversal symmetry is broken by a spatially inhomogeneous magnetic field with zero average, and the Hall conductance again equals the Chern number of the band [2]. Six years later in another striking development Haldane answered the second question showing by an explicit construction of a tight-binding model on a honeycomb lattice that a quantized Hall conductance can arise from a fully filled band even in the absence of a net magnetic field [2].

A Landau level involves a set of exactly degenerate single-particle states and thus, at a fractional filling, the kinetic energy alone does not select a ground state, but instead, it falls to the interactions to force the issue [2]. By contrast, a Chern band typically will have a significant dispersion that will select a unique kinetic-energy-dominated ground state at reasonable interaction strengths, as it does in all metals. Recognizing this, all three paper [4, 5, 6] devote considerable effort to constructing lattice models with nearly flat (degenerate) Chern bands. The authors of article [5] construct a flattened version of Haldane's model on a square lattice. They note that while a fully flattened model requires the inclusion of electron hopping over arbitrarily large distances, the hopping amplitudes decrease exponentially, which allows a relatively flat band to be constructed by keeping a small set of hopping amplitudes. The relevant flatness parameter, which should be large for the effects of interactions to be important, is the ratio of the band gap (which sets a bound on the strength of the interactions one can safely include) to the bandwidth and they show how to get this number up to seven with just second-neighbor interactions. Similarly the authors of articles [4, 6] construct models on the kagome and checker-board lattices, which also exhibit large values of the flatness parameter [2].

With a flat Chern band in hand, the authors of the paper [5] introduce interactions and study the system at a fractional filling of 1/3 through numerical computation on a sized system. They find two of the classic signatures of the 1/3 FQHE state: a fractional quantum Hall conductance that was close to the filling fraction, and a nontrivial ground-state degeneracy with periodic boundary conditions. As a test, they vary the band structure continuously to a topologically trivial band and find that these features go away. In a related piece of unpublished work, another group finds similar results at filling of 1/3 and 1/5 [2]. Altogether, this work offers strong evidence that fractionally filled Chern bands do indeed exhibit the FQHE [2].

In this earlier work, the authors [2] studied a fixed filling factor while varying the flux per plaquette from small values and large unit cells, where the standard Landau level description holds, to somewhat larger flux values and smaller unit cells, where that description broke down. As they were able to change this parameter without any evidence of encountering a phase transition, the latter limit constituted an observation of the FQHE in the presence of strong lattice effects [2].
5 Incommensurately modulated sinusoidal polarization waves \( Q_T, Q_P, Q_b \) in stationary Schrödinger boundary problem for domain wall for data storage of \( \text{PmBaMn}_2\text{O}_6 \) nanosize ferroics

We have found the following Landau-free energy expansion for specify domain walls for data storage of thin \( \text{PmBaMn}_2\text{O}_6 \) nanofilms with the \( P4mm \) space group of reference data [14, 168]

\[
F_1 = \frac{1}{2} \alpha_t Q_T^2 + \frac{1}{4} \beta_t Q_T^4 + \frac{1}{2} \alpha_s (s_1^2 + s_2^2) + \frac{1}{2} \beta_s (s_1^2 + s_2^2)^2 + \frac{1}{4} \alpha_s Q_b^2 + \frac{1}{2} \alpha_p Q_b^2 + F_{\text{ext}} + F_{\text{bs}} + F_{\text{bpb}},
\]

where \( F_{\text{ext}} = \delta_{\text{etz}} Q_T^2 s_1 s_2, F_{\text{bs}} = \delta_{\text{bs}} Q_b (s_1^2 - s_2^2) \), as well as \( F_{\text{bpb}} = \delta_{\text{bpb}} Q_T Q_b Q_P \).

The Euler-Lagrange dynamic equations for the order parameter component \( Q_T(k,z) \) further denoted as \( f_{\alpha}(k,z) \equiv \phi_\alpha(k,z) \) we have specified from the Lagrange function of ferroic film by varying of variables of the Functional Eq. (55)

\[
\begin{align*}
\alpha_t Q_T + \beta_t Q_T^3 + \delta_{\text{etz}} s_1 s_2 + \delta_{\text{bpb}} Q_T Q_P = 0, \\
\alpha_s Q_b + \delta_{\text{bs}} (s_1^2 - s_2^2) + \delta_{\text{bpb}} Q_T Q_P = 0, \\
\alpha_p Q_b + \delta_{\text{bpb}} Q_T Q_P = 0,
\end{align*}
\]

where \( s_1^2 + s_2^2 = Q_T^2, s_1 = 0.88 Q_T, s_2 = 0.47 Q_T, Q_T^2 + Q_P^2 + Q_b^2 + Q_s^2 = 1 \).

From equations system Eqs. (56) one can find

\[
\alpha_p Q_p + \delta_{\text{bpb}} Q_T Q_b = 0,
\]

the order parameter \( Q_b \)

\[
Q_b = -\frac{\alpha_p Q_p}{\delta_{\text{bpb}} Q_T},
\]

and substituting \( Q_b \) in Eqs. (56) we have derived

\[
-\frac{\alpha_p Q_p}{\delta_{\text{bpb}} Q_T} + \delta_{\text{bpb}} Q_T Q_P + \delta_{\text{bs}} (s_1^2 - s_2^2) = 0,
\]

as well as

\[
-\alpha_p \alpha_s Q_P + \delta_{\text{bpb}} Q_T^2 Q_P + \delta_{\text{bpb}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T = 0,
\]

and after elementary transformations one can find

\[
Q_P (-\alpha_p \alpha_s + \delta_{\text{bpb}} Q_T^2) = \delta_{\text{bs}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T.
\]

Hence we have found

\[
Q_p = \frac{\delta_{\text{bs}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T}{(-\alpha_p \alpha_s + \delta_{\text{bpb}} Q_T^2)},
\]

as well as

\[
Q_b = -\alpha_p \frac{\delta_{\text{bs}} (s_1^2 - s_2^2)}{(-\alpha_p \alpha_s + \delta_{\text{bpb}} Q_T^2)}.
\]

Substituting Eqs. (62), (63) into Eqs. (56) we have specified the equation for \( Q_T \) order parameter as follows

\[
\alpha_t Q_T + \beta_t Q_T^3 + \delta_{\text{etz}} s_1 s_2 - \frac{\alpha_p \delta_{\text{bpb}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T}{(-\alpha_p \alpha_s + \delta_{\text{bpb}} Q_T^2)} = 0,
\]

after elementary transformation we have obtained

\[
(\alpha_t Q_T + \beta_t Q_T^3 + \delta_{\text{etz}} s_1 s_2) (-\alpha_p \alpha_s + \delta_{\text{bpb}} Q_T^2)^2 - \alpha_p \delta_{\text{bpb}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T = 0,
\]

as well as

\[
(\alpha_t Q_T + \beta_t Q_T^3 + \delta_{\text{etz}} s_1 s_2) (\alpha_p \delta_{\text{bpb}} Q_T^2 + \delta_{\text{bpb}} Q_T^4 - \alpha_p \delta_{\text{bpb}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T = 0,
\]

and hence

\[
\alpha_p \alpha_p \alpha_p Q_T^2 - 2 \alpha_p \alpha_p \alpha_p \delta_{\text{bpb}} Q_T^2 + \alpha_p \delta_{\text{bpb}} Q_T^4 + \delta_{\text{bpb}} Q_T^4 - 2 \alpha_p \alpha_p \alpha_p \delta_{\text{bpb}} Q_T^2 + \beta_t Q_T^3 + \delta_{\text{bpb}} Q_T^4 + \delta_{\text{bpb}} Q_T^4 + \alpha_p \delta_{\text{bpb}} (s_1^2 - s_2^2) \delta_{\text{bpb}} Q_T = 0.
\]

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Let us known a ferroic film of thickness $h$ confined in the $z$ direction in the region $z \in [-h/2, h/2]$, hence $k_z \rightarrow -i \frac{\pi}{h}$.

The film surfaces are covered by two ideally conducting soft electrodes, which do not affect its mechanical state and we have specified the nanofilm by the wave vectors in the space with orthonormal basis $[1, \zeta_v]$ like as

$$
\tilde{\psi}_{\zeta_v, k_z} = |v\zeta_v, k_z| = \left[\sum_{m=1}^{\infty} \psi_k^{(1)}[i, v] \psi_v(Z) \right]_{[1, \zeta_v]} \left[2, \zeta_v \right],
$$

(68)

where $\zeta_v$ is a chirality, $\psi_v(Z) = \sqrt{\frac{1}{h}} \sin(\pi m(\frac{Z}{h} + \frac{1}{2})) = \sqrt{\frac{1}{h}} \sin(\pi mZ), Z = (\frac{Z}{h} + \frac{1}{2})$.

We have specified the depolarization field $E_d^f[\mathbf{f}_1(x, z)]$ by Eq. (69) and which has been involved into Schrödinger boundary problem Eqs. (82), (83) determines the chirality form of solution of Eqs. (82), (83).

$$
E_d^f[\tilde{Q}_T(k, z)] = \left\{ \frac{\tilde{Q}_T(k, z)}{c_T}, \frac{\tilde{Q}_T(k, z)^*}{c_T} \right\}.
$$

(69)

In the simplest one-component one-dimensional case considered hereinafter the bulk part of the free energy $F$ which depends on one-component order parameter $Q_T$ coupled with the order parameters $Q_\iota$ and $Q_\nu$ which depend only on the coordinate $x$ and their gradients has the following form

$$
F \approx \int_0^h dz \int_{-\infty}^{\infty} dxdy \left\{ \frac{1}{2} \alpha_t Q_T^2 + \frac{1}{2} \beta_t Q_T^4 + \frac{1}{2} \beta_s (s_1^2 + s_2^2) + \frac{1}{2} \gamma_s s_1 s_2 + \frac{1}{2} \alpha_s Q_s^2 + \frac{1}{2} \alpha_p Q_p^2 + F_{\iota\iota} + F_{\iota\nu} + F_{\nu\nu} - Q_T \left( \frac{\Delta T}{\gamma} + E_{\nu} \right) + \frac{1}{2} \frac{\partial Q_T}{\partial s_1^2} + \frac{1}{2} Q_T \left( \frac{\partial Q_T}{\partial s_1} \right)^2 + \frac{1}{2} \int_0^\infty dxdy(\frac{\partial^2 Q_T}{\partial x_1^2}) \right\},
$$

(70)

where

$$
Q_T = \delta_{s\alpha}(s_1^2 - s_2^2),
$$

(71)

$$
Q_s = -\alpha_s \frac{\epsilon_{s\alpha}(s_1^2 - s_2^2)}{\epsilon_{s\alpha}(s_1^2 + s_2^2)},
$$

(72)

as well as $\alpha_t = \alpha(T) - \frac{(Q_{\iota\iota} + Q_{\iota\nu}) h}{s_{1\iota} + s_{1\nu}} - 2 \frac{(Q_{\iota\iota} + Q_{\nu\nu}) h}{s_{1\iota} + s_{1\nu}} \frac{2h_0}{c_T}, \beta_t = \beta + 2 \frac{(Q_{\iota\iota} + Q_{\nu\nu}) h}{s_{1\iota} + s_{1\nu}}, s_{ij}$ is an elastic coefficients. Hence we have specified the stationary Schrödinger boundary problem in space confined medium or planar waveguide as follows

$$
\Gamma \frac{\partial Q_T}{\partial T} + \alpha^* Q_T + \alpha^* Q_T^2 + \beta^* Q_T^4 + \beta'^* Q_T^4 + \gamma' Q_T^2 + \gamma' Q_T^4 - g_5 \frac{\partial Q_T}{\partial x} - (g_1 + v_1 Q_T)^2 \frac{\partial Q_T}{\partial x} \left( \frac{\partial Q_T}{\partial x} \right)^2 + \delta_{s1s2} \alpha_s^2 \frac{\partial^2 Q_T}{\partial x^2} = E_d^f(x, y, z) + E_0 \exp(i \omega t),
$$

(73)

where

$$
\alpha^* = \alpha_t \alpha_s^2 \alpha_p - \alpha_p \frac{\partial}{\partial s_1} (s_1^2 - s_2^2)^2 \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(74)

$$
\alpha'^* = -\alpha_p \alpha_s \frac{\partial s_1}{\partial s_1} \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(75)

$$
\beta'^* = -\alpha_t \alpha_s \frac{\partial s_1}{\partial s_1} \frac{\partial^2}{\partial s_1^2} p_{\iota\iota} + \alpha_s^2 \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(76)

$$
\beta'^* = \delta_{s1s2} \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(77)

$$
\gamma'^* = \alpha_s \frac{\partial^2}{\partial s_1^2} p_{\iota\iota} - 2 \alpha_p \alpha_s \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(78)

$$
\gamma'^* = \delta_{s1s2} \frac{\partial^2}{\partial s_1^2} p_{\iota\iota},
$$

(79)

and with boundary conditions

$$
(\alpha^* Q_T - g_5 \frac{\partial Q_T}{\partial x}) - w_1 \frac{\partial Q_T}{\partial x} - v_1 Q_T \frac{\partial Q_T}{\partial x} \left( Q_T + \frac{\partial Q_T}{\partial x} \right) \big|_{s_1=0} = 0,
$$

(80)

$$
(\alpha^* Q_T + g_5 \frac{\partial Q_T}{\partial x}) + w_1 \frac{\partial Q_T}{\partial x} + v_1 Q_T \frac{\partial Q_T}{\partial x} \left( Q_T + \frac{\partial Q_T}{\partial x} \right) \big|_{s_1=h} = 0.
$$
The solutions of the stationary Schrödinger boundary problem in space confined medium or planar waveguide in the form of expansion into a series

\[ p(k, z, t) = \sum_{n=1}^{\infty} \psi_{n}^{(1)}(k, t) \psi_{n}(Z) \exp(-i\lambda_{n}(k) \frac{z}{\hbar}) + E_{n}(k, \omega) \frac{f(k, z) \exp(i\omega t)}{\lambda_{n}(k) + \omega} \]  

(81)

Euler-Lagrange equations which linearized with respect to the polarization and displacement fluctuations acquire the following form in two-dimensional \( k_{i} = \sqrt{k_{x}^{2} + k_{y}^{2}} \) space

\[ \left[ a^{*} - g_{3} \frac{\partial^{2}}{\partial z^{2}} + g_{1} k_{i}^{2} + w_{1} k_{i}^{2} \right] f_{n}(k_{z}, z) - E_{n}^{2} f_{n}(k_{z}, z) = -\delta_{i, 1} s_{1} s_{2} \alpha_{1}^{2} \alpha_{2}^{2}, \]  

(82)

The Euler-Lagrange dynamic equations for the order parameter component \( Q_{\Gamma}(k_{1}, z) \) further denoted as \( f_{n}(k_{1}, z) \equiv \phi_{\Gamma}^{(1)}(z, k_{1}) \) we have specified from the Lagrange function of ferrofilm by varying of variables of the Functional. The boundary conditions for \( f_{n}(k_{1}, z) \) have the form

\[ \left( a^{*} f_{n} - g_{3} \frac{\partial^{2} f_{n}}{\partial z^{2}} - w_{1} \frac{\partial f_{n}}{\partial z} + v_{1} f_{n} \frac{\partial f_{n}}{\partial z} + v_{2} f_{n} \frac{\partial f_{n}}{\partial z} \right) \bigg|_{z=0} = 0, \]

\[ \left( a^{*} f_{n} + g_{3} \frac{\partial^{2} f_{n}}{\partial z^{2}} + w_{1} \frac{\partial f_{n}}{\partial z} - v_{1} f_{n} \frac{\partial f_{n}}{\partial z} + v_{2} f_{n} \frac{\partial f_{n}}{\partial z} \right) \bigg|_{z=\alpha} = 0 \]  

(83)

where \( f_{n}(k_{1}, z) \equiv \phi_{\Gamma}^{(1)}(z, k_{1}) = \sum_{n=1}^{m} \psi_{n}^{(1)}(n, v) \psi_{n}(Z), j = 1, 2 \) where also \( \alpha = \nu \) from [13] i.e. \( \psi_{n}^{(1)}(n, \nu) \psi_{n}(Z) = (\nu) \).

The solutions of Schrödinger equations can be found in the form Eq. (68) i.e. \( \phi_{\Gamma}^{(1)(\nu)}(z, k_{1}) = \sum_{n=1}^{m} \psi_{n}^{(1)}(n, \nu) \psi_{n}(Z), j = 1, 2 \) by premultiplying the equations system of Eq. (82) on \( \psi_{n}(Z) \) functions and by integrating the later equations system on quantum box boundaries.

Let us known the following integral relations for the found solutions algebra equation systems

\[ \int_{0}^{1} h \psi(n, Z) \psi(n, Z) dZ = 1, \]  

(84)

\[ \int_{0}^{1} h \psi(n, Z) \psi(k, Z) dZ = 0, \]  

(85)

if \( n \neq k \),

\[ \int_{0}^{1} \psi(n, Z) \frac{d^{2}}{dz^{2}} \psi(n, Z) dZ = \frac{\lambda_{n}^{2} - (1 - \frac{1}{2})}{\lambda_{n}^{2}}, \]  

(86)

if \( n = k \),

\[ \int_{0}^{1} \psi(n, Z) \frac{d^{2}}{dz^{2}} \psi(n, Z) dZ = 0, \]  

(87)

\[ \int_{0}^{1} \psi(n, Z) \frac{d^{2}}{dz^{2}} \psi(n, Z) dZ = -\frac{n^{2}}{\hbar}, \]  

(88)

Since

\[ \int_{0}^{1} \psi(n, Z) \frac{d^{2}}{dz^{2}} \psi(n, Z) dZ = -\frac{n^{2}}{\hbar}, \]  

(89)

then we have replaced \( \lambda_{n}^{2} \rightarrow -\frac{n^{2} \pi^{2}}{\hbar^{2}} \) in Eq. (82), keeping in the mind that the solutions of Schrödinger equations can be found in the form Eq. (68) i.e. \( \phi_{\Gamma}^{(1)(\nu)}(z, k_{1}) = \sum_{n=1}^{m} \psi_{n}^{(1)}(n, \nu) \psi_{n}(Z), j = 1, 2 \) by premultiplying the equations system of Eq. (82) and hence the depolarization fields Eq. (69) on \( \psi_{n}(Z) \) functions and by integrating the later equations system on quantum box boundaries one can find

\[ \int_{0}^{1} \sin(\pi n Z) \sin(\pi n Z) dZ = 0, \]  

(90)

\[ \int_{0}^{1} \sin(\pi n Z) \sin(\pi n Z - \frac{1}{2}) \sin(\pi m Z) dZ = 0, \]  

(91)

\[ \int_{0}^{1} \sin(\pi n Z) \sin(\pi n Z - \frac{1}{2}) \sin(\pi m Z) dZ = \frac{2 \sin(\pi n Z) \sin(\pi m Z)}{k^{2} m^{2} \pi^{2}}, \]  

(92)

\[ \int_{0}^{1} \sin(\pi n Z) \sin(\pi n Z - \frac{1}{2}) \sin(\pi m Z) dZ = \frac{2 \sin(\pi n Z) \sin(\pi m Z)}{k^{2} m^{2} \pi^{2}}, \]  

(93)

\[ \psi_{n}(Z) \sim \sin(\pi n Z) - \frac{\pi^{2} \sin(\pi n Z)}{\pi^{2} \sin(\pi m Z)} \sin(\pi m Z), \]  

(94)

with the following approximation of matching of quasi-classical functions.
we have found the quasi-exactly stationary Schrödinger boundary problem for wave vectors \( \Psi_{k}^{(j)}[n,v] \) of quantized band energy dispersions

\[
\Psi_{k}^{(j)}[n,v] = \left( \alpha - 2 \gamma (Q_T)^2 + 4 \beta (Q_T)^3 + 7 \gamma (Q_T)^5 \right) \delta_{nm},
\]

where \( h = 1, \nu = 1 \) do not coincide with \( n, \alpha' = \alpha + 2 \gamma (Q_T)^2 + 4 \beta (Q_T)^3 + 7 \gamma (Q_T)^5 \), and \( \langle Q_T \rangle \) is the average order parameter (for a bulk single domain sample the averaged order parameter \( \langle Q_T \rangle = (\sqrt{\beta^2 - 4 \alpha' \gamma' - \beta^2})/2 \gamma' \) [13]. We have specified the depolarization field \( E_{b}^{(f)}(k_{y},z) \) by Eq. (69) and which has been involved into Schrödinger boundary problem Eqs. (82), (83) determines the chirality form of solution of Eqs. (82), (83). Hence in addition inherently true the following dispersion expressions for star of wave vector

\[
s_1 = 0.88 + \nu, s_2 = 0.47 + \nu, \quad s_3 = 0.47 + \nu, s_4 = 0.88 + \nu, s_5 = 0.47 + \nu, \quad s_6 = 0.88 + \nu,
\]

as well as \( s_2 = 0.47 + \nu, s_4 = 0.88 + \nu \), as well as \( s_2 = 0.47 + \nu, s_4 = 0.88 + \nu \), and the dispersion region \( \Gamma_{1}^{1} \) has been found as \( s_1 = 0.88 + \nu, s_2 = 0.47 + \nu, q_{3} = 0.88 + \nu \), where \( q_{3} = k_{t} \).

I think in the article [13] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovskaya et al. were not allowed and consequently the quantized band energies were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96).

<table>
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<th>Symbol and dimension</th>
<th>Curie-Weiss constant</th>
<th>Curie temperature</th>
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<tr>
<td>( \alpha_{f} \times 10^{6} \text{Jm}/(\text{C} \cdot \text{K}) )</td>
<td>1.6</td>
<td>( T_{C} = 193 )</td>
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<td>( T_{C} ) (K)</td>
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<th>Symbol and dimension</th>
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<th>LGD-coefficient at ( \eta^{6} )</th>
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<tr>
<td>( \beta \times 10^{3} \text{JC}^{-4} \text{m}^{5} )</td>
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<th>Symbol and dimension</th>
<th>Gradient coefficient at ( \langle \eta \rangle^{2} )</th>
<th>Material permittivity</th>
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<tr>
<td>( g_{1} \times 10^{10} \text{C}^{-2} \text{m}^{3} \text{J} )</td>
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<td>( \epsilon_{33} = 11 )</td>
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<td>( \epsilon_{0} )</td>
<td>9.38</td>
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</table>

<table>
<thead>
<tr>
<th>Symbol and dimension</th>
<th>Material parameters in Landau free-energy expansion for PmBaMn( <em>{2} \text{O}</em>{6} ) [14].</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{e} \times \text{eV/Å}^{2} )</td>
<td>-0.29</td>
</tr>
<tr>
<td>( \alpha_{b} \times \text{eV/Å}^{2} )</td>
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<td>( \alpha_{p} \times \text{eV/Å}^{2} )</td>
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<td>( \beta_{e} \times \text{eV/Å}^{2} )</td>
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<td>( \delta_{111} \times \text{eV/Å}^{3} )</td>
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<td>( \delta_{100} \times \text{eV/Å}^{3} )</td>
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<tr>
<td>( \delta_{110} \times \text{eV/Å}^{3} )</td>
<td>-0.26</td>
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</table>

In Figs. 3-5 we have presented the computation solutions of quantized normalized charges densities of the Eq. (96) stationary Schrödinger boundary problem for different thickness \( h \) of PmBaMn\( _{2} \text{O}_{6} \) ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition in the form of quantized hysteresis loops.

Keeping in mind [7] the fixed potential at the top gate \( \phi_{0}(x,z = -d) = V_{c} \) and zero potential at the bottom gate \( \phi_{EF}(x,z = h) = 0 \) the continuity of the electric potential at the graphene layer \( \phi_{0}(x,z = 0) = \phi_{EF}(x,z = 0) \) and the
Figure 3. (Color online) Normalized surface charge densities versus transverse wave vector in nanosize ferroic PmBaMn$_2$O$_6$ with width $h = 50$ nm.

Figure 4. (Color online) Normalized surface charge densities versus transverse wave vector in nanosize ferroic PmBaMn$_2$O$_6$ with width $h = 20$ nm.
equivalence of the difference of the electric displacement normal compound to the surface charge density $\sigma(x)$ in graphene $D^E_2(x, z = 0) - D^E_3(x, z = 0) = \sigma(x)$. The latter condition we have specified for single ferroelectric domain wall (FDW) at contact with graphene as [7]

$$-\varepsilon_0 \varepsilon_3 \frac{\partial \phi(z)}{\partial z} + e_0 \varepsilon_3 \frac{\partial \phi(z)}{\partial z} \mp Q_T(x_{\pi}^z 0, z = 0) = \sigma_{n,2}(x), \quad (97)$$

$$-\varepsilon_0 \varepsilon_3 \frac{\partial \phi(z)}{\partial z} + e_0 \varepsilon_3 \frac{\partial \phi(z)}{\partial z} \mp Q_P(x_{\pi}^z 0, z = 0) = \sigma_{n,2}(x), \quad (98)$$

where $Q_T$, $Q_P$ are order parameters in Landau-free energy expansion for specify domain walls for data storage of thin PmBaMn$_2$O$_6$ nanofilms with the $P4mm$ space group of reference data [9].

We have found that the approximate analytical expressions for the values $\sigma_{n,2}$ and $\sigma_{n,2}^{(1)}$ [7]

$$\sigma_{n,2}(T, u_m, k) \approx \varepsilon_0 V_g(\frac{32}{4} + \varepsilon_3^{(1)} T) \pm Q_T(k), \quad (99)$$

$$\sigma_{n,2}^{(1)}(T, u_m, k) \approx \varepsilon_0 V_g(\frac{32}{4} + \varepsilon_3^{(1)} T) \pm Q_P(k), \quad (100)$$

where $u_m$ is an elastic strain in the film, $u_m^e(h) \approx u_m \frac{h}{h_{sd}}$, $h_{sd}$ is the thickness of misfit dislocations, $d$ is the thickness of the oxide dielectric layer and $h$ is the thickness of the FE film.

The unique band spectrum of graphene leads to the unconventional integer quantum Hall effect (IQHE) [7]. It is known [7] that the xy-component of conductance tensor $\sigma_{xy}$ via Dirac-like spectrum of graphene and consequently additional double degeneration of a zero Landau level (LL) which is common for conduction and valence cones result in a special form of a Hall quantization $\sigma_{xy} = -\frac{\pi}{2\pi} \nu$, $\nu = \pm 2(2k + 1)$, where $\nu$ is a number of edge modes, $k = 0, 1, 2, ...$ is an integer number. The inner spin-valley symmetry in graphene leads to four-fold degeneracy of each LL but LL with $k = 0$ has an additional double degeneracy. Nonzero $k$ is given by the expression $k = [\frac{\mu}{4n} - \frac{1}{2}]$, where symbol $[\ ]$ stands for the integer part of a number, $n$ is the 2D concentration of electrons in the graphene channel and $n_B = \frac{eB}{2\pi h}$ is the density of the magnetic field flux, threading the 2D surface corresponding to the degree of the k-th LL occupation. In the article [7] have been explained theoretically the peculiarities of IQHE observed experimentally in graphene with p-n junction across the conduction channel.

Hence the FDW induces a p-n junction in graphene at that the wall plane coincides with the p-n junction position [7].

The hallmark [8] of the quantum anomalous Hall (QAH) phase is the existence of chiral edge states, i.e. gapless channels at the edge propagating unidirectionally. The edge spectrum in the QAH phase with Chern number $\theta = -1$ confirms existence of one chiral channel per edge [8]. Because [8] the chirality of the edge channel is determined by the sign of Chern number the two QAH phases with $\theta = \pm 1$ must have edge channels propagating i opposite directions.
Consequently by tuning across the topological phase transition (via Weil Hall state (WHS)) between the two QAH phases one can switch the propagating direction of the edge channel [8]. This can be easily detected with a standard electric transport measurement [8].

Since \( \mathcal{S} \) is broken in the 2D WHS state the Chern number \( \mathcal{C} = \int_{\mathcal{BZ}} \Omega(\mathbf{k})d\mathbf{k} \), where \( \Omega(\mathbf{q}) = -2\Im\langle \partial_\mathbf{q} u_+|\partial_\mathbf{q} u_- \rangle \) which is the integral of Berry curvature over the BZ is typically nonzero [8]. Hence because \( \Omega \) is an even function under inversion when the two Weil points are connected by \( \mathcal{S} \) the two valley topological charges must be identical and we should have \( \mathcal{C} = 1 \) or \( \mathcal{C} = -1 \) [8].

We have investigated the transition from 2D WHS to QAH phases. Because the Weil points in 2D WHS are protected \( \mathcal{C}/\mathcal{C}_{\text{cal}} \) should have breaking can be easily achieved by slightly rotating magnetization vector away from the ground-state orientation [8].

### 6 The self-consistent stationarity quantized solutions of non-linear Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory

The polar-active properties of ferroelectric films have been calculated in Landau-Ginsburg-Devonshire theory framework [25, 23, 24, 13] with account of polarization vector gradients and surface charges and dipole layers on the interface in the conditions of stability of spontaneous polarization. Under the electric voltage feeds on the electrode or sound of scanning electron microscope the polarization redistributions have been achieved success. Under the increasing the electric voltage the local reversal of polarizations [25, 23, 24, 13] has been achieved which one lied to the creation the intergrown domains in thin films. Under domains recording the velocity of changes of extrinsic electric field \( \mathbf{E} \) was negligibly small and without loss of accuracy one can suppose \( \text{rot}\mathbf{E} \approx 0 \). From Maxwell equations

\[
\text{rot}\mathbf{E} = \frac{\partial\mathbf{B}}{\partial t},
\]

if we have assumed that the magnetic induction \( \mathbf{B} \) in Maxwell equation was independent quantity with respect to time then

\[
\text{rot}\mathbf{E} \approx 0.
\]

Let us known for the vector of electric field strength following expressions

\[
\mathbf{E} = -\nabla \varphi,
\]

as well as for the vector of electric-flux density the following Maxwell equations

\[
\text{div}\mathbf{D} = \text{div}(\mathbf{E} + 4\pi\mathbf{P}) = \frac{4\pi}{\varepsilon_0} \rho(\varphi),
\]

where \( \rho(\varphi) \) is charges density, \( \varepsilon_0 \) is dielectric constant.

The ferroelectric film that occupies the region \( -L < z < 0 \) is transversally isotropic i.e. permittivity \( \varepsilon_{11} = \varepsilon_{22} \) at zero electric field. We assume that the dependence of plane polarization components on \( E_{1,2} \) can be linearized as \( P_{1,2} \approx \varepsilon_0(\varepsilon_{11} - 1)E_{1,2} \) \( \varepsilon_0 \) is the universal dielectric constant) while the polarization component \( P_3 \) nonlinear depends on external fields. The spontaneous polarization \( P_3 \) is directed along the polar axis \( z \). Further we assume that the dependence of polarization vector acquires the form [23, 24, 13]

\[
\mathbf{P}(\mathbf{r}) = \left[ \varepsilon_0(\varepsilon_{11} - 1)E_1, \varepsilon_0(\varepsilon_{11} - 1)E_2, P_3(\mathbf{E}, \mathbf{r}) + \varepsilon_0(\varepsilon_{33}^b - 1)E_3 \right].
\]

Let us known the quasiequilibrium polarization distribution \( P_3(x, y, z) \) with taking into account the gyrotropy effects which are shown to be connected with the stationary non-linear Schrödinger equation of domain wall found from Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory (in the articles [23, 24, 13] we have found the space solution of Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory only) as follows

\[
i\alpha \frac{\partial P_3}{\partial t} - g(\Delta t + \frac{\partial^2}{\partial z^2})P_3 + \beta |P_3|^2 = -\frac{\partial \varphi}{\partial z}.
\]

with taking into consideration for the vector of electric field strength the following expressions \( E_z = -\frac{\partial \varphi}{\partial z} \), with the following boundary conditions

\[
(P_3 + \lambda_1 \frac{\partial P_3}{\partial z})_{z=0} = -P_b,
\]

\[
\text{div}\mathbf{D} = \text{div}(\mathbf{E} + 4\pi\mathbf{P}) = \frac{4\pi}{\varepsilon_0} \rho(\varphi),
\]

where \( \rho(\varphi) \) is charges density, \( \varepsilon_0 \) is dielectric constant. The ferroelectric film that occupies the region \( -L < z < 0 \) is transversally isotropic i.e. permittivity \( \varepsilon_{11} = \varepsilon_{22} \) at zero electric field. We assume that the dependence of plane polarization components on \( E_{1,2} \) can be linearized as \( P_{1,2} \approx \varepsilon_0(\varepsilon_{11} - 1)E_{1,2} \) \( \varepsilon_0 \) is the universal dielectric constant) while the polarization component \( P_3 \) nonlinear depends on external fields. The spontaneous polarization \( P_3 \) is directed along the polar axis \( z \). Further we assume that the dependence of polarization vector acquires the form [23, 24, 13]

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\mathbf{P}(\mathbf{r}) = \left[ \varepsilon_0(\varepsilon_{11} - 1)E_1, \varepsilon_0(\varepsilon_{11} - 1)E_2, P_3(\mathbf{E}, \mathbf{r}) + \varepsilon_0(\varepsilon_{33}^b - 1)E_3 \right].
\]

Let us known the quasiequilibrium polarization distribution \( P_3(x, y, z) \) with taking into account the gyrotropy effects which are shown to be connected with the stationary non-linear Schrödinger equation of domain wall found from Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory (in the articles [23, 24, 13] we have found the space solution of Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory only) as follows

\[
i\alpha \frac{\partial P_3}{\partial t} - g(\Delta t + \frac{\partial^2}{\partial z^2})P_3 + \beta |P_3|^2 = -\frac{\partial \varphi}{\partial z}.
\]

with taking into consideration for the vector of electric field strength the following expressions \( E_z = -\frac{\partial \varphi}{\partial z} \), with the following boundary conditions

\[
(P_3 + \lambda_1 \frac{\partial P_3}{\partial z})_{z=0} = -P_b,
\]
Quantized Polarization

where we introduce $\Delta_1 = \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}$ Laplace operator. Inhomogeneity $P_b$ describes the effects of interface polarization stemming from the interface bonding effect and associated interface dipole. Hence the translation symmetry breaking inevitably in the vicinity of the any interface will give rise to inhomogeneity in the boundary conditions in Eqs. (106), (107), (108). The equations Eqs. (104), (106), (107), (108) yield the coupled system

$$
\left(\frac{\partial^2 \varphi}{\partial z^2} + \Delta_1 \varphi\right) = 0, \quad -H < z < -L,
$$

$$
\epsilon_0 \epsilon_{11} \Delta_1 \varphi = -\rho_f(\varphi), \quad -L < z < 0,
$$

where $\epsilon_{11}$ is background insulator permittivity of incipient ferroelectric, $\epsilon_j$ is the semiconductor bare lattice permittivity. The equations Eqs. (109), (110), (111) were supplemented with the boundary conditions $\varphi(x, y, -L - H) = U_0(x, y)$, $\varphi(x, y, -L + 0) = \varphi(x, y, -L - 0)$, $\varphi(x, y, z \to \infty) = 0$, $\varphi(x, y, z) = \varphi(x, y, -L) = U_0(x, y)$, $\epsilon_0 \epsilon_{11} \frac{\partial \varphi(x, y, -L)}{\partial z} = \sigma_f(x, y)$, $-L < z < 0$.

For the space charge density inside the doped p-type (or n-type) semi-infinite semiconductor has the form $\rho_1(\varphi) = q[p(\varphi) + \rho_0^H(\varphi)]$, $\rho(\varphi) = N_a^p F(\frac{\varphi - E_a - E_F}{k_B T})$, $N_a^p(\varphi) = N_a^p F(\frac{\varphi - E_a - E_F}{k_B T})$, $n(\varphi) = N_a^p F(\frac{\varphi - E_a - E_F}{k_B T})$, $N_a^p(\varphi) = N_a^p F(\frac{\varphi - E_a - E_F}{k_B T})$, $F(\theta) = (\exp^0 + 1)^{-1}$, is the Fermi-Dirac distribution function, $q$ is carriers charge, $E_F$, $E_a$, $E_g$, $E_c$ are the energies of Fermi level, valence band, conduction band, and donor and acceptor levels in the quasineutral region of the semiconductor correspondingly. We have assumed $p(0) + N_a^p(0) = 0$, $n(0) = N_a^p \approx \text{const}$, $N_a^p \approx \text{const}$, and Boltzmann approximation for electrons $E_c - E_F - q\varphi >> k_B T$ or holes $E_c - E_F + q\varphi >> k_B T$ lead to expressions $\rho_1 \approx q\rho_0^p(\varphi \approx \frac{E_F}{k_B T} - 1)$, $\rho_1 \approx qn_0^p(\varphi \approx \frac{E_F}{k_B T} - 1)$ correspondingly where $\rho_0^p$ and $n_0^p$ are equilibrium concentrations of holes and electrons in the quasineutral region of the semiconductor.

Hence in abrupt junction approximation the space charge density near the interface of the strongly doped p-type (or n-type) semi-infinite semiconductor has the form $\rho_1(\varphi) = q\rho_0^p - \frac{\rho_0^p}{|z| < W_i}$, where we assume the charge density $\rho_0^0 = q\rho_0^0$ with depth $W_i = W_{sp}$ (or $\rho_0^0 = qn_0^0$ with depth $W_i = W_{in}$). We have supposed that the ferroelectric film is wide-gap semiconductor or insulator. Hence $\rho_f(\varphi) \approx 0$ at region $-L < z < 0$. Keeping in mind the above found boundary conditions the one-dimension solutions of Eqs. (109), (110), (111) have the form

$$
\varphi(z) = -\frac{\rho_0^0}{2\epsilon_0 \epsilon_{11}} (W_i - z)^2 \theta(W_i - z), \quad z > 0,
$$
Hence we have found the electric field after the following elementary algebraic transformations we have found \( \theta \) where

\[
\varphi(z) = U_e + \frac{H}{e_\sigma e_{33}} (\sigma_z + \sigma_f - \rho^0_{33} W_z), \quad -L \leq z < 0,
\]

\[
\varphi(z) = U_e + \frac{H + U_z}{e_\sigma e_{33}} (\sigma_z + \sigma_f - \rho^0_{33} W_z), \quad -L - H \leq z < -L,
\]

where \( \theta(z) \) is the Hevithaide step function.

Hence we have found the electric field \( E_3 \) with \( E_3(z) = -\frac{\partial \varphi}{\partial z} \) and electrical displacement \( D_3 \) distributions based on Eqs. (112), (113), (114) in the form

\[
E_3(z) = -\frac{P_0(z)}{e_0 e_{33}} + \frac{\rho^0_{33} W_z - \sigma_z}{e_0 e_{33}},
\]

\[
D_3(z) = \rho^0_{33} W_z - \sigma_z, \quad -L < z < 0,
\]

\[
E_3(z) = \frac{\rho^0_{33}}{2e_0 e_{33}} (z - W_z) \theta(W_z - z), \quad z > 0,
\]

\[
D_3(z) = \rho^0_{33} (z - W_z) \theta(W_z - z), \quad z > 0,
\]

\[
P_3(z) = \rho^0_{33} \frac{z - W_z}{2} \theta(W_z - z), \quad \sigma_z \geq 0.
\]

Hence the compensated free charge \(-\sigma_f = D_3(-L)\) has been created at second interface \( z = -L \), and the electroneutral condition has the form \(-\sigma_z + \rho^0_{33} W_z + \sigma_f = 0\). Hence we have found the self-consistent stationary soliton solution of non-linear Schrödinger or Euler-Lagrange boundary problem for the polarization vector distributions with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory as following

\[
\alpha P_3 + \beta P_3^3 - g(\Delta_t + \frac{\partial^2}{\partial z^2})P_3 = -\frac{\partial \varphi}{\partial z},
\]

Under variate Eq. (120) we have found

\[
\alpha \frac{\partial P_3}{\partial z} + 3\beta P_3 \frac{\partial P_3}{\partial z} - g \Delta_t \frac{\partial P_3}{\partial z} - g \frac{\partial^2}{\partial z^2} P_3 = -\frac{\partial^2 \varphi}{\partial z^2},
\]

with the Maxwell equation in thin polar active ferroelectric film

\[
\epsilon^b_{33} \frac{\partial^2 \varphi}{\partial z^2} + \epsilon^f_{11} \Delta_t \varphi = \frac{1}{e_0} (\frac{\partial P_3}{\partial z} - \rho_f(\varphi)), \quad -L < z < 0,
\]

after the following elementary algebraic transformations we have found

\[
-\frac{\partial^2 \varphi}{\partial z^2} = \frac{1}{e_3} \Delta_t \varphi - \frac{1}{e_0 e_{33}} (\frac{\partial P_3}{\partial z} - \rho_f(\varphi)), \quad -L < z < 0,
\]
as well as
\[
\frac{\partial P_3}{\partial z} (\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) - g \frac{\partial^2 P_3}{\partial z^2} - \rho_i(\phi) = 0.
\]  
(124)
Hence under integrate transformations of Eq. (124) we have written
\[
\int dz \frac{\partial P_3}{\partial z} (\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) - \frac{1}{L} \int_0^L dz \int dz \frac{\partial^2 P_3}{\partial z^2} + \rho_i(\phi) = 0.
\]  
(125)
under the second integrate transformation of Eq. (125) we have written
\[
P_3(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) - g \frac{1}{L} \int_0^L dz \int dz \frac{\partial^2 P_3}{\partial z^2} - \rho_i(\phi) = 0.
\]  
(126)
under the third integrate transformation of Eq. (126) we have derived
\[
P_3(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) - g \frac{1}{L} \int_0^L dz \int dz \frac{\partial^2 P_3}{\partial z^2} - \rho_i(\phi) = 0,
\]  
(127)
the equation from which one can find polarization \(P_3\) via average polarization \(\langle P_3\rangle\) in the form
\[
P_3(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) - g \frac{1}{L} \int_0^L dz \int dz \frac{\partial^2 P_3}{\partial z^2} - \rho_i(\phi) = 0.
\]  
(128)
Keeping in mind the found boundary condition we have found
\[
P_3 = \frac{\psi(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) + C_1}{\rho_i(\phi) = 0},
\]  
(129)
as well as
\[
P_3 = \frac{\phi(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) + C_1}{\rho_i(\phi) = 0},
\]  
(130)
for the coupled states of particles from the conditions that the first quantized wave function or first quantized polarization \(P_3\) has not the node on quantum well width and keeping in mind the next boundary conditions we have found
\[
\psi = A \xi \sinh(\xi), \quad \psi_1 = A \lambda_1 \cosh(\xi), \quad \phi = A \xi \cos(\xi), \quad \phi_1 = -A \lambda_2 \sinh(\xi), \quad \xi = \sqrt{\varepsilon \sigma_{33}^5} \frac{\hbar}{E_1},
\]  
(131)
the following trial wave functions and the following normalization condition for wave function with boundary conditions in the form
\[
\frac{d}{dz} \int_0^L d \xi (\xi \sinh(\xi) + \lambda_1 \cosh(\xi) + \xi \cos(\xi) - \lambda_2 \sinh(\xi))^2 = 1
\]  
hence from the normalization condition for wave vector A\(\xi (\xi^2 + \lambda_1 \lambda_2 \sinh(2\xi) + \xi (\lambda_1 + \lambda_2) \cos(\xi))^2 = 1\) we have found the two constants of integrations \(\psi(z, L) = 1 - A \xi \cos(\xi) - \lambda_1 \cosh(\xi) + \xi \cos(\xi) - \lambda_2 \cosh(\xi)^2)\), and \(\phi(z, L) = A^2 \xi (\xi \sinh(\xi^2) + \lambda_1 \cosh(\xi) + \xi \sinh(\xi^2) + \lambda_2 \cosh(\xi^2))\) for the first quantized polarization \(P_3\)
\[
P_3^{(\text{first quantized polarization})} = \frac{\psi(\alpha + 3\beta (P_3)^2 + \frac{1}{\varepsilon \sigma_{33}^5}) + C_1}{\rho_i(\phi) = 0},
\]  
(132)
\(P_3\) as well as the quantized energies.

For the average polarization \(P_3\) we have written the following expression
\[
\langle P_3 \rangle = \rho_0 W_1 - \sigma_j + \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2 = \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} (1 + \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2),
\]  
(133)
where \(z = \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2\), and for the average polarization \(P_3\) we can derive the cubic equation like
\[
\langle P_3 \rangle = \rho_0 W_1 - \sigma_j + \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2 = \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} (1 + \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2),
\]  
(134)
where for average magnitudes one can find the next expressions
\[
\langle P_3 \rangle = \rho_0 W_1(1 + \frac{e_h \rho_0}{2e_\sigma_\varepsilon_{33}^5} W_2),
\]  
(135)
then the cubic equation for average polarization \(P_3\) we can derive as follows
The quantized out-of-plane active ferroelectric nanofilm with semiconductor with boundary parameters:

\[ E_b^f(L, H) = \frac{\epsilon_b^f(\psi)}{\epsilon_b^f+\epsilon_b^h}U_b + \frac{\alpha_b}{\epsilon_b^f}P_b(\phi), \]

are built-in electrostatic field and the external field correspondingly. From the equation

\[ (\alpha + \frac{1}{\epsilon_b^f (1 - \frac{\epsilon_b^f L(\psi)}{\epsilon_b^f+\epsilon_b^h})})(P_3) + (P_3)^3(3\beta - g(\psi)) + 3\beta P_b(\phi) (P_3)^2 = E_b^f(L, H) + E_b^f(L, H), \tag{134} \]

where \( E_b^f(L, H) = \frac{\epsilon_b^f(\psi)}{\epsilon_b^f+\epsilon_b^h}U_b + \frac{\alpha_b}{\epsilon_b^f}P_b(\phi) \)

For the proper ferroelectrics the coefficient \( \alpha(T) = \alpha_b(T - T^c) \), where \( T \) is an absolute temperature, \( T^c \) is the Curie temperature renormalized by the epitaxial misfit strain \( u_m \). The strain originates from the thin film (a) and the semiconductor (c) lattice constants mismatch [100, 102, 66, 93, 83, 125, 124].

7 The self-consistent 5-polinomial quantized solutions of stationary Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory

Let us known the quasiequilibrium polarization distribution \( P_3(x, y, z) \) with taking into account the gyrotrony effects which are shown to be connected with the stationary non-linear Schrödinger equation of domain wall found from Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory (in the articles [23, 24, 13] we have found the space solution of Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory only)

\[ \left( \frac{\partial^2 P_3}{\partial x^2} + \beta P_3|P_3|^2 + \gamma P_3(|P_3|^2)^2 - g(\Delta_1 + \frac{\partial^2}{\partial z^2})P_3 = -\frac{\partial P_3}{\partial z}, \right. \]

with taking into consideration for the vector of electric field strength the following expressions

\[ E_z = -\frac{\partial P_3}{\partial z} \]

where we introduce \( \Delta_1 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \) Laplace operator. Inhomogeneity \( P_b \) describes the effects of interface polarization stemming from the interface bonding effect and associated interface dipole. Hence the translation symmetry breaking inevitably in the vicinity of the any interface will give rise to inhomogeneity in the boundary conditions in Eqs. (136), (137), (138). The equations Eqs. (104), (136), (137), (138) yield the coupled system

\[ (\frac{\partial^2 \psi}{\partial z^2} + \Delta_1(\psi) = 0, \quad -H - L < z < -L, \]

\[ \epsilon_b^b \frac{\partial^2 \psi}{\partial z^2} + \epsilon_1 \Delta_1 \psi = \frac{1}{\epsilon_0} (\frac{\partial P_3}{\partial z} - \rho_1(\psi)), \quad -L < z < 0, \]

\[ \epsilon_0 \epsilon_b^f (\frac{\partial^2 \psi}{\partial z^2} + \Delta_1 \psi) = -\rho_1(\psi), \quad z > 0, \]
Table 4. The material parameters: Band gap, (eV), Carriers concentrations cm$^{-3}$, Background permittivity, LGD-expansion coefficient for ferroelectrics, electrostriction, and elastic constants [23, 24, 13, 119].

<table>
<thead>
<tr>
<th>Material</th>
<th>Band gap, (eV)</th>
<th>Concentrations cm$^{-3}$</th>
<th>LGD-expansion coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaSrMnO$_3$ (LSMO) half-metal</td>
<td>1 p type</td>
<td>1.83×10$^{22}$</td>
<td>Permittivity: $\varepsilon_\alpha = 30$ (effective lattice constant $a = 3.876$ Å)</td>
</tr>
</tbody>
</table>
| BiFeO$_3$ (BFO) ferroelectric | 3 | Wide band-gap semiconductor | Nonferroelectric $e_{33}^b = 9.0$ | $a_{hex} = 5.58$ Å, $\alpha_T = 2.3 \times 10^5$ m/(FK), $T_c = 1098 - 1103$ K, $T = 300$ K, $\beta_b = -2.44 \times 10^9$ m$^5$/(C$^2$F), $\gamma = 1.56 \times 10^8$ m$^2$/(C$^2$F), $P_s = 0.8$, (C/m$^2$), $g = 10^{-8} - 10^{-9}$, (m$^3$/F), $g = 1.3 \times 10^{-8}$, (m$^3$/F), $Q_{11} = 0.032$, (m$^4$/C$^2$), $Q_{12} = -0.016$, (m$^4$/C$^2$), $Q_{44} = 0.01$, (m$^4$/C$^2$), $s_{11} = 5.29 \times 10^{-12}$, (m$^2$/N), $s_{12} = -1.85 \times 10^{-12}$, (m$^2$/N), $s_{44} = 1.47 \times 10^{-12}$, (m$^2$/N), $\varepsilon_{33}^b = 43$, $\varepsilon_{33}^b = 5.7$, $a = 0.3905$ Å, $\alpha_T = 1.26 \times 10^6$ m/(FK), $T_c = 38$ K, $T_a = 84$ K, $\beta_b = 8.1 - 6.8 \times 10^6$ m$^5$/(C$^2$F), $\gamma = 4 \times 10^{-12}$ m$^6$/(C$^2$F), $g = 10^{-8} - 10^{-9}$, (m$^3$/F), $g = 0.99 \times 10^{-9}$, (m$^3$/F), $Q_{11} = 0.051$, (m$^4$/C$^2$), $Q_{12} = -0.016$, (m$^4$/C$^2$), $Q_{44} = 0.020$, (m$^4$/C$^2$), $s_{11} = 3.89 \times 10^{-12}$, (m$^2$/N), $s_{12} = -1.06 \times 10^{-12}$, (m$^2$/N), $s_{44} = 8.20 \times 10^{-12}$, (m$^2$/N), $\varepsilon_{33}^b$ is background insulator permittivity of incipient ferroelectric, $\varepsilon_\alpha$ is the semiconductor bare lattice permittivity. The equations Eqs. (139), (140), (141) were supplemented with the boundary conditions $\varphi(x,y,-L+0) = \varphi(x,y,-L-0)$, $\varphi(x,y,\infty) = 0$, $\varphi(x,y,+0) - \varphi(x,y,-0) = U_b$, $e_{33}^b \frac{\partial \varphi(x,y,\infty)}{\partial x} = P_3(x,y,-0)$, $e_{33}^b \frac{\partial \varphi(x,y,\infty)}{\partial y} = \sigma_\alpha(x,y)$, $e_{33}^b \frac{\partial \varphi(x,y,L+0)}{\partial x} = P_3(x,y,-L+0)$, $e_{33}^b \frac{\partial \varphi(x,y,L+0)}{\partial y} = \sigma_\alpha(x,y)$, $e_{33}^b \frac{\partial \varphi(x,y,-L+0)}{\partial x} = P_3(x,y,-L+0)$, $e_{33}^b \frac{\partial \varphi(x,y,-L+0)}{\partial y} = \sigma_\alpha(x,y)$, where $U_b$ is the contact potential difference at the insulator-semiconductor interface, $\varepsilon_{33}^b$ is the dielectric constant of the dielectric gap between the tip and ferroelectric surface. We have assumed that polarization vector $P_3$ is directed along $z$ axis that corresponds to the plane electrodes.
Table 5. The renormalization of $T_{\text{Curie}}^*$ by misfit strain and dislocations originated at critical film thickness $h_d$: $T_{\text{Curie}}^*$ at $L \leq h_d$, $T_{\text{Curie}}^*$ at $L > h_d$, $T_{\text{Curie}} + \frac{h_d}{a_0} \frac{\partial \Omega}{\partial \xi} + T_{\text{Curie}}^*$ at $L > h_d$, $T_{\text{Curie}} + \frac{h_d}{a_0} \frac{\partial \Omega}{\partial \xi}$ [23, 24, 13, 119].

<table>
<thead>
<tr>
<th>Interface orientation</th>
<th>$u_m$, nm</th>
<th>$h_d$, nm</th>
<th>$T_{\text{Curie}}^*$ at $L \leq h_d$</th>
<th>$T_{\text{Curie}}^*$ at $L &gt; h_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSMO/STO [100]</td>
<td>-0.0074</td>
<td>10.8</td>
<td>$-12 T_{\text{Curie}}$</td>
<td>$T_{\text{Curie}}(1 - \frac{h_d}{L})$</td>
</tr>
<tr>
<td>LSMO/BFO [100]</td>
<td>-0.0230</td>
<td>2.4</td>
<td>$0.6 T_{\text{Curie}}$</td>
<td>$T_{\text{Curie}}(1 - \frac{0.4 h_d}{L})$</td>
</tr>
</tbody>
</table>

Hence we have found the self-consistent stationarity quantized solution of 5-polynomial Schrödinger or Euler-Lagrange boundary problem for the polarization vector distributions with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory as following

$$\alpha P_3 + \beta P_3^3 + \gamma P_3^5 - g(\Delta \xi + \frac{\partial^2 \xi}{\partial \xi^2})P_3 = -\frac{\partial^2 \xi}{\partial \xi^2},$$  \hspace{1cm} (142)

Under variate Eq. (142) we have found

$$\alpha \frac{\partial P_3}{\partial z} + 3 \beta P_3^2 \frac{\partial P_3}{\partial z} + 5 \gamma P_3^4 \frac{\partial P_3}{\partial z} - g \Delta \xi \frac{\partial P_3}{\partial z} - g \frac{\partial^3 P_3}{\partial z^3}P_3 = -\frac{\partial^3 \phi}{\partial z^3},$$  \hspace{1cm} (143)

with the Maxwell equation in thin polar active ferroelectric film

$$e_{33}^b \frac{\partial^2 \varphi}{\partial z^2} + e_{11}^\alpha \Delta \xi \varphi = \frac{1}{e_0} \left[ \frac{\partial P_3}{\partial z} - \rho_f(\varphi) \right], \quad L < z < 0,$$  \hspace{1cm} (144)

after the following elementary algebraic transformations we have found

$$-\frac{\partial^2 \varphi}{\partial z^2} = \frac{e_{33}^b}{e_0} \Delta \xi \varphi - \frac{1}{e_0 e_{33}^b} \left( \frac{\partial P_3}{\partial z} - \rho_f(\varphi) \right), \quad L < z < 0,$$  \hspace{1cm} (145)

as well as

$$\frac{\partial P_3}{\partial z} (\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b}) - g \frac{\partial^3 P_3}{\partial z^3} = 0.$$  \hspace{1cm} (146)

Hence under integrate transformations of Eq. (146) we have found

$$\int dz \frac{\partial P_3}{\partial z} (\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b}) - g \frac{\partial^3 P_3}{\partial z^3} = \int_0^L \frac{d\rho_f(\varphi)}{e_{33}^b} = C_1 = 0,$$  \hspace{1cm} (147)

under the second integrate transformation of Eq. (147) we have written

$$P_3 (\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b}) - g \frac{\partial^3 P_3}{\partial z^3} = \int_0^L d\rho_f(\varphi) = C_1 = 0,$$  \hspace{1cm} (148)

under the third integrate transformation of Eq. (148) we have derived

$$P_3 (\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b}) - g \frac{\partial^3 P_3}{\partial z^3} = \int_0^L d\rho_f(\varphi) = C_1 = 0,$$  \hspace{1cm} (149)

the equation from which one can find polarization $P_3$ via average polarization $\langle P_3 \rangle$ in the form

$$P_3 (\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b}) - g \langle P_3 \rangle^3 - \frac{\rho_f(\varphi)}{e_{33}^b} = C_1 = 0.$$  \hspace{1cm} (150)

Keeping in mind the found boundary condition we have found

$$P_3 = \frac{\rho_f(\varphi)}{(\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b})} + C_1,$$  \hspace{1cm} (151)

as well as

$$P_3 = \frac{\rho_f(\varphi)}{(\alpha + 3 \beta (P_3)^2 + 5 \gamma (P_3)^4 + \frac{1}{e_0 e_{33}^b})} + C_1,$$  \hspace{1cm} (152)

for the coupled states of particles from the conditions that the first quantized wave function or first quantized polarization $P_3$ has not the node on quantum well width and keeping in mind the next boundary conditions we have found

$$\psi = \xi \sinh(\xi), \quad \psi_1 = \lambda_1 \cosh(\xi), \quad \phi = \xi \cosh(\xi), \quad \phi_1 = -\lambda_2 \sinh(\xi),$$  \hspace{1cm} (136, 145, 133, 134, 135, 136, 168, 17, 64, 141, 140),

the following trial wave functions and the following normalization condition for wave function with boundary conditions in the form

$$\frac{\partial^2 \xi}{\partial z^2} = \frac{\partial^2 \xi}{\partial z^2} + \lambda_1 \cosh(\xi) + \xi \cosh(\xi) - \lambda_2 \sinh(\xi)^2 = 1,$$  \hspace{1cm} (149)

hence from the normalization condition for wave vector $A^2(\xi^2 + \lambda_1 \lambda_2 \sinh(2\xi) + \xi(\lambda_1 + \lambda_2) \cosh(2\xi)) + 1$ we have
found the two constants of integrations \( \psi(z, L) = 1 - A^2 \xi(-\xi \sinh(\frac{\xi}{L}) - \lambda_1 \cosh(\frac{\xi}{L}) + \lambda_2 \cosh(\frac{\xi+1}{L})) \), and \( \phi(z, L) = A^2 \xi(\xi \sinh(\frac{\xi+1}{L}) + \lambda_2 \cosh(\frac{\xi+1}{L})) \) for the first quantized polarization \( P_3 \)

\[
p_{3,\text{first quantized polarization}} = \frac{g \varepsilon c_{3s}^2 (P_3)^3 + \rho W_{s} - \sigma_i}{(\varepsilon c_{3s}^2 + \rho W_{s} - \sigma_i)} \psi(z, L) - \frac{h}{\varepsilon c_{3s}^2} \phi(z, L),
\]

(153)

\[
p_{3} = \frac{g \varepsilon c_{3s}^2 (P_3)^3 + \rho W_{s} - \sigma_i}{(\varepsilon c_{3s}^2 + \rho W_{s} - \sigma_i)} \psi(z, L) - \frac{h}{\varepsilon c_{3s}^2} \phi(z, L),
\]

(154)

where \( \psi(z, L) = 1 - A^2 \xi(-\xi \sinh(\frac{\xi}{L}) - \lambda_1 \cosh(\frac{\xi}{L}) + \lambda_2 \cosh(\frac{\xi+1}{L})) \), and \( \phi(z, L) = A^2 \xi(\xi \sinh(\frac{\xi+1}{L}) + \lambda_2 \cosh(\frac{\xi+1}{L})) \).

For the average polarization \( P_3 \) we have written the following expression

\[
\langle P_3 \rangle = \rho_s W_s - \sigma_i + \frac{\varepsilon c_{3s}^2}{2L} W_s^2 - \frac{\varepsilon c_{3s}^2}{L} (U_s + U_c) + \frac{h}{\varepsilon c_{3s}^2} (\sigma_j + \sigma_f - \rho_i W_s),
\]

(155)

where \( z = \frac{c_{3s}^2}{2L} W_s \), and for the average polarization \( P_3 \) we can derive the 5-polynomial equation like

\[
(\alpha + \frac{1}{\varepsilon c_{3s}^2}) \langle P_3 \rangle + (P_3)^2 (3\beta - g(\psi)) + 3\beta P_3 \langle \phi \rangle \langle P_3 \rangle^2 + 5\gamma P_3 \langle \phi \rangle \langle P_3 \rangle^4 + 5\gamma \langle P_3 \rangle^5 = \frac{\varepsilon c_{3s}^2 W_s - \sigma_i}{\varepsilon c_{3s}^2} \langle \psi \rangle - \frac{h}{\varepsilon c_{3s}^2} \langle \phi \rangle (\alpha + \frac{1}{\varepsilon c_{3s}^2}),
\]

(156)

where for average magnitudes one can find the next expressions \( \langle \psi \rangle = 1 - \frac{\varepsilon c_{3s}^2 (2\xi + \lambda_1 + \lambda_2)}{2\xi (\xi + \lambda_1 + \lambda_2 + \lambda_2)} \), \( \langle \phi \rangle = \frac{\varepsilon c_{3s}^2}{2\xi (\xi + \lambda_1 + \lambda_2)} \).

8 Summary

In the article we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize PmBaMnO\(_4\) ferroics with incommensurately modulated sinuosoidal polarization waves \( Q_r \), \( Q_p \), \( Q_h \). The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for \( Q_r \), \( Q_p \), \( Q_h \) with sinuosoidal depolarization waves for the thin PmBaMnO\(_4\) nanofilm are specified. We have specified the analytical quantized solutions for the band energy dispersion in nanosize ferroics in framework of Landau-Ginsburg-Devonshire (LGD) theory as well as the numerical computer calculation results for the thin PmBaMnO\(_4\) nanofilm have been presented. We have presented the computation solutions of quantized normalized charges densities of the Eq. (96) stationary Schrödinger boundary problem for different thickness \( h \) of PmBaMnO\(_4\) ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition in the form of quantized hysteresis loops. We have found the quantized out-of-plane polarizations \( P_3 \) dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilms with semiconductor with different boundary parameters. The quantized out-of-plane \( P_3 \) polarizations were clearly visible like the festive fires of candles. I think in the article [13] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska e.t.al. were not allowed and consequently quantized energies or wave vectors were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96).

References


