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Fractional Quantum Hall Effect at researches of nanosize and Landau quantized quasiparticle solutions of Schrödinger boundary problem of domain wall for data storage of thin SmBaMn$_2$O$_6$ nanofilms at contact with graphene channel of p-n junction with magnetic field

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Abstract In the article we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize PmBaMn$_2$O$_6$ as well as SmBaMn$_2$O$_6$ ferroic in magnetic field applied in Landau symmetric gauge with incommensurately modulated sinusoidal polarization waves $Q_T$, $Q_P$, $Q_b$. The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for $Q_T$, $Q_P$, $Q_b$ with sinusoidal depolarization waves for the thin PmBaMn$_2$O$_6$ as well as SmBaMn$_2$O$_6$ in magnetic field applied in symmetric gauge ferroic nanofilms are specified. We have presented the computation solutions of quantized normalized charges densities of the Eq. (130) stationary Schrödinger boundary problem for different thickness $h$ of PmBaMn$_2$O$_6$ as well as SmBaMn$_2$O$_6$ in magnetic field applied in symmetric gauge ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition and applicable that Fractional Quantum Hall Effect (FQHE) in the form of nanosize quantized as well as Landau quantized hysteresis loops that were clearly visible like Fairy-tale Phoenix. We have found the quantized out-of-plane polarizations $P_3$ dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilm with semiconductor with different boundary parameters. The thin Os$_x$Ra$_{1-x}$FeO$_3$ nanofilms have quasiparticle excitations that can be described by (2+1)-dimensional Dirac theory from developed a four-sublattice model (FSM) for the analytical description of A-cation displacements in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type. We have found that FSM explains the coexistence of rhombohedral (R), orthorhombic (O) and spatially modulated phases observed by atomic-resolution scanning transmission electron microscopy (STEM) in Os-doped RaFeO$_3$. Using this approach we atomically resolve the theoretical model of the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in Os$_x$Ra$_{1-x}$FeO$_3$ polymorphs. We have shown that this produces an unconventional form of the quantized Hall conductivity $\sigma_{xy} = -(2e^2/h)(2n+1)$ with $n = 0, 1, 2, ...$ This unconventional quantization is caused by the analytical chirality of $n = 0$ Landau level.

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Spin Hall insulators have connected with respect to dissipationless spin transport via a sample with the spin-orbit interactions (SOI) effects. The extremum rings of the valence band in tensile strained zincblende GaN quantum well as well as the two valley shape of the valence band in GaN nanotube grown along c axis were a collection have attracted our attention both for their fundamental studies and for their device applications. The Quantum Spin Hall effects are shown to be related with intraband transitions of bulk GaN. In the framework of the effective mass theories we have solved the Schrödinger equation if the topological insulator transformation is achieved. The exact solutions of the Schrödinger equations as well as Quantum Spin Hall effect of intraband transitions of bulk GaN are found. For the hexagonal symmetry of GaN the Effective Hamiltonian based on C_{6v} point symmetry group was found. In the article for Quantum Hall effect of intraband phototransitions of bulk GaN the expressions of Berry curvature as well as Hall conductivities have been found when the topological insulator transformation is achieved. Spin Hall insulators have connected with the Topological insulator (TI) as well as Exciton insulators. The dissipationless spin transport via the sample are shown to be related with Spin Hall insulator self-consistent solution of the Schrödinger equations for electrons and holes and the Poisson equations at the presence of spatially varying quantum well potential due to the piezoelectric effect and the local exchange-correlation potential in ZnO quantum well. In ZnO/Zn,MgO quantum well the electron-hole pairing leads to the exciton insulator states. An exciton insulator states with a gap 3.4 eV of ZnO quantum well and at 310 K temperature are predicted. If the electron and hole are separated, their energy is higher on 0.2 meV than if they are paired. The particle-hole pairing leads to the Cooper instability.
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2 Introduction

The computation solutions of order parameters of ferroelectric phase transition and applicable that Fractional Quantum Hall Effect (FQHE) in the form of nanosize quantized as well as Landau quantized hysteresis loops that were clearly visible like Fairy-tale Phoenix [1, 2, 3]. The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for $Q_T$, $Q_P$, $Q_b$ with sinusoidal depolarization waves for the thin PmBaMn$_2$O$_6$ as well as SmBaMn$_2$O$_6$ in magnetic field applied in symmetric gauge ferroelectric nanofilms are specified. We have presented the computation solutions of quantized normalized charges densities of the Eq. (130) stationary Schrödinger boundary problem for different thickness $h$ of PmBaMn$_2$O$_6$ as well as SmBaMn$_2$O$_6$ in magnetic field applied in symmetric gauge ferroelectric (FE) nanofilms. The thin Os$_x$Ra$_{1-x}$FeO$_3$ nanofilms have quasiparticle excitations that can be described by $(2+1)$-dimensional Dirac theory from developed a four-sublattice model (FSM) for the analytical description of A-cation displacements in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type. We have found that FSM explains the coexistence of rhombohedral (R), orthorhombic (O) and spatially modulated phases observed by atomic-resolution scanning transmission electron microscopy (STEM) in Os-doped RaFeO$_3$. Using this approach we atomically resolve the theoretical model of the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in Os$_x$Ra$_{1-x}$FeO$_3$ polymorphs. We have shown that this produces an unconventional form of the quantized Hall conductivity $\sigma_{xy} = -(2e^2/h)(2n + 1)$ with $n = 0, 1, 2, .. [1, 2, 3]$.

In the article [4, 5, 6, 7, 8] the authors study theoretically the properties of buckled graphene-like materials such as silicene and germanene in a strong perpendicular magnetic field and a periodic potential. The authors analyze how the spin-orbit interaction and the perpendicular electric field influence the energy spectra of these systems [4, 5, 6, 7, 8]. When the magnetic flux through a unit cell of the periodic potential measured in the magnetic flux quantum is a rational
number $\alpha = p/q$, then in each Landau level the energy spectra have a band structure which is characterized by the corresponding gaps. The authors study the dependence of those gaps on the parameters of the buckled graphene-like materials [4, 5, 6, 7, 8]. Although some gaps have weak dependence on the magnitude of the spin-orbit coupling and the external external electric field, there are gaps that show strong nonmonotonic dependence on these parameters [4, 5, 6, 7, 8]. For $\alpha = 1/2$ the spin-orbit interaction also opens up a gap at one of the Landau levels [4, 5, 6, 7, 8]. The magnitude of the gap increases with spin-orbit coupling and decreases with the applied electric field [4, 5, 6, 7, 8]. The authors [4, 5, 6, 7, 8] highlight the differences and similarities between monolayer and bilayer graphene and focus on thermodynamic properties such as the compressibility the plasmon spectra the weak localization correction quantum Hall effect and optical properties. Confinement of electrons in graphene is nontrivial due to Klein tunnelling [4, 5, 6, 7, 8].

The experimental discovery of an anomalous integer quantum Hall effect in graphene has enabled the study of a correlated two-dimensional electronic system in which the interacting electrons behave like massless chiral fermions [9, 10]. In the article [9] the authors report the observation of the fractional quantum Hall effect in ultraclean suspended graphene. In addition the author show that at low carrier density graphene becomes an insulator with a magnetic-field-tunable energy gap [9]. These newly discovered quantum states offer the opportunity to study correlated Dirac fermions in the presence of large magnetic fields [9].

The article [11] unveils a mapping between a quantum fractal that describes a physical phenomena and an abstract geometrical fractal. The quantum fractal is the Hofstadter butterfly discovered in an iconic condensed matter problem of electrons moving in a two-dimensional lattice in a transverse magnetic field [11]. In the Hofstadter butterfly these integers encode the topological quantum numbers of quantum Hall conductivity [11]. In the article [12] the universal phase and field configuration properties are determined by topological features related to the Hopf invariant $Q$ and its generalizations. For sufficiently low densities a ring-shaped density distribution may be favored over stripes. For an $L < Q$ phase a gain in free energy in the two-component Ginsburg-Landau model occurs when a phase transition to a nonuniform current state occurs [12].

The free energy expansion in powers of order parameters is employed in Landau-Ginsburg-Devonshire (LGD) free energy and can be used in the phase-field modeling of macro and nanosized ferroelectrics [13]. Understanding of ferroic behavior at surfaces, interfaces, and defects as well as the nature of ferroelectric states considerably advanced in the last decades with the advancement of scanning transmission electron microscopy (STEM) [13]. Probing the unit-cell level symmetry breaking via STEM allowed the determination of direct atomic positions, from which the spatial distributions of order parameters fields can be mapped [13]. Hence here we derive a model LGD-type free energy describing directly observable degrees of freedom available from atomic-resolution STEM [13]. We have proposed a theoretical four-sublattice model (FSM) for analytical description of cation displacement in (anti)ferroelectric-antiferrodistortive perovskites of $ABO_3$ type, that explain the coexistence of rhombohedral (R), orthorhombic (O), and spatially modulated (SM) phases observed by atomic-resolution STEM [13]. Using this approach we atomically resolve and theoretically model the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in perovskites of Os$_x$Ra$_{1-x}$FeO$_3$ polymorphs with magnetic field. Hence for over 50 years of researches of ferroic materials the properties of these materials were explored using the combination of scattering techniques that provides the information on the nature and symmetry of order parameters and macroscopic measurements that provided the information on the corresponding expansion coefficients and the nature of phase transitions [13].

In this earlier work, the authors [14] studied a fixed filling factor while varying the flux per plaquette from small values and large unit cells, where the standard Landau level description holds, to somewhat larger flux values and smaller unit cells, where that description broke down. As they were able to change this parameter without any evidence of encountering a phase transition, the latter limit constituted an observation of the FQHE in the presence of strong lattice effects [14].

In a landmark paper in 1982, Thouless, Kohmoto, Nightingale, and den Nijs [15] analyzed the uniform-field Hall effect in a strong periodic potential that was known to lead to an intricate spectrum, the so-called Hofstadter butterfly (see Fig. 2); they showed that it gives rise to an integer QHE under certain conditions, i.e., whenever the chemical potential lies in a gap [14]. Indeed, the Hall conductance was shown to map to a topological invariant associated with filled bands - the (first) Chern number. In this model, time-reversal symmetry is broken by a spatially inhomogeneous magnetic field with zero average, and the Hall conductance again equals the Chern number of the band [14]. Six years later in another striking development Haldane answered the second question showing by an explicit construction of a tight-binding model on a honeycomb lattice that a quantized Hall conductance can arise from a fully filled band even in the absence of a net magnetic field [14].

A Landau level involves a set of exactly degenerate single-particle states and thus, at a fractional filling, the kinetic energy alone does not select a ground state, but instead, it falls to the interactions to force the issue [14]. By contrast, a Chern band typically will have a significant dispersion that will select a unique kinetic-energy-dominated ground state at reasonable interaction strengths, as it does in all metals. Recognizing this, all three paper [16, 17, 18] devote considerable effort to constructing lattice models with nearly flat (degenerate) Chern bands. The authors of article [17] construct a flattened version of Haldane’s model on a square lattice. They note that while a fully flattened model requires the inclusion of electron hopping over arbitrarily large distances, the hopping amplitudes decrease...
exponentially, which allows a relatively flat band to be constructed by keeping a small set of hopping amplitudes. The relevant flatness parameter, which should be large for the effects of interactions to be important, is the ratio of the band gap (which sets a bound on the strength of the interactions one can safely include) to the bandwidth and they show how to get this number up to seven with just second-neighbor interactions. Similarly the authors of articles [16, 18] construct models on the kagome and checker-board lattices, which also exhibit large values of the flatness parameters [14].

With a flat Chern band in hand, the authors of the paper [17] introduce interactions and study the system at a fractional filling of 1/3 through numerical computation on a sized system. They find two of the classic signatures of the 1/3 FQHE state: a fractional quantum Hall conductance that was close to the filling fraction, and a nontrivial ground-state degeneracy with periodic boundary conditions. As a test, they vary the band structure continuously to a topologically trivial band and find that these features go away. In a related piece of unpublished work, another group finds similar results at filling of 1/3 and 1/5 [14]. Altogether, this work offers strong evidence that fractionally filled Chern bands do indeed exhibit the FQHE [14].

The unique band spectrum of graphene as well as 2D Weil PI3Cl, PI3Br, PI3, PII3, RuCl3 and Group-VI Dichalcogenides materials leads to the unconventional integer quantum Hall effect (IQHE) [19, 20]. It is known [19] that the xy-component of conductance tensor \( \sigma_{xy} \) via Dirac-like spectrum of graphene and consequently additional double degeneration of a zero Landau level (LL) which is common for conduction and valence cones result in a special form of a Hall quantization \( \sigma_{xy} = -\frac{e^2}{2\pi^2} v, v = \pm 2(2k + 1) \), where \( v \) is a number of edge modes, \( k = 0, 1, 2, ... \) is an integer number. The inner spin-valley symmetry in graphene leads to four-fold degeneracy of each LL but LL with \( k = 0 \) has an additional double degeneracy. Nonzero \( k \) is given by the expression \( k = \left\lfloor \frac{a}{2\pi n_B} \right\rfloor \), where symbol \( \lfloor \cdot \rfloor \) stands for the integer part of a number, \( n \) is the 2D concentration of electrons in the graphene channel and \( n_B = \frac{eB}{2\pi} \) is the density of the magnetic field flux, threading the 2D surface corresponding to the degree of the k-th LL occupation. In the article [19] have been explained theoretically the peculiarities of IQHE observed experimentally in graphene with p-n junction across the conduction channel.

Hence the FDW induces a p-n junction in graphene at that the wall plane coincides with the p-n junction position [19]. The hallmark [20] of the quantum anomalous Hall (QAH) phase is the existence of chiral edge states, i.e. gapless channels at the edge propagating unidirectionally. The edge spectrum in the QAH phase with Chern number \( \chi = -1 \) confirms existence of one chiral channel per edge [20]. Because [20] the chirality of the edge channel is determined by the sign of Chern number the two QAH phases with \( \chi = \pm 1 \) must have edge channels propagating in opposite directions. Consequently by tuning across the topological phase transition (via Weil Hall state (WHHS)) between the two QAH phases one can switch the propagating direction of the edge channel [20]. This can be easily detected with a standard electric transport measurement [20].

Since \( \mathcal{F} \) is broken in the 2D WHS state the Chern number \( \chi = \int_{BZ} \Omega(k)dk \), where \( \Omega(q) = -23(\partial_{u_l} u_r, \partial_{u_l} u_r) \) which is the integral of Berry curvature over the BZ is typically nonzero [20]. Hence because \( \Omega \) is an even function under inversion when the two Weil points are connected by \( \mathcal{F} \) the two valley topological charges must be identical and we should have \( \chi = 1 \) or \( \chi = -1 \) [20].

We have investigated the transition from 2D WHS to QAH phases. Because the Weil points in 2D WHS are protected by the mirror \( M_y \) breaking \( M_x \), will generally remove the Weil points and open the energy gap [20]. This symmetry breaking can be easily achieved by slightly rotating magnetization vector away from the ground-state orientation [20]. In the article [1, 21, 22, 23] we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize ferroics with incommensurately modulated sinusoidal polarization waves \( P_y \). The both quantized energies and wave vectors of stationary Schrödinger boundary problem for \( P_y \) with sinusoidal depolarization waves are specified. We have specified the analytical quantized solutions for the frequency dispersion of soft phonon modes in nanosize ferroics in framework of Landau-Ginsburg-Devonshire (LGD) theory as well as the numerical calculation results for the thin SrTiO3 nanofilm have been presented. The frequency quantized dispersion of soft phonon modes in nanosize ferroics found from stationary Schrödinger boundary problem for domain wall are shown to be related with the Rashba and Dresselhaus quantized energies as well as their wave vectors for electrons in quantized magnetic field. The quantized polarization as well as elastic displacement of stationary Schrödinger boundary problem for domain wall for data storage based on bulk ferroics are found. We have found the quantized solutions of stationarity non-linear high-polynomial Schrödinger or Euler-Lagrange boundary problem for the order parameter as well as elastic displacement in nanosize ferroics. Quantized solutions of one-dimensional Maxwell equations for thin ferroelectric films have been specified which were bonded with stationary solution of cubic and 5-polynomial Schrödinger equation are found. I think in the article [24] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska et.al. were not allowed and consequently were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96).

The quantized polarization as well as energy levels solutions of stationary Schrödinger boundary problem for domain wall for data storage based on interface of polar-active ferroelectric nanofilm with semiconductor are found. The self-consistent stationarity quantized solutions of the both like cubic thus 5-polynomial Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin fer-
roelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire (LGD) theory theory were presented. We have found the quantized solutions of stationarity non-linear high-polynomial Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor. Quantized solutions of one-dimensional Maxwell equations for thin ferroelectric films have been specified which were bonded with stationary solution of cubic and 5-polynomial Schrödinger equation are found. Recently one can see the increasing attentions regarding complex superstructures with incommensurate nature. The optica of the complex superstructures with incommensurate nature has been intensively explored. A particular examples for those complex superstructures were a modulated structures, a structures of charge density waves or spin density waves, an Abrikosov vortex lattice in the crimping films of the superconducting aluminium in the perpendicular magnetic field in which arise the interference between the vortex period caused the magnetic field and the crimping period [1, 21, 22, 23]. To sum up problem of Quantum Field Theory is the instability of incommensurate phase of Abrikosov vortex lattice of graphene and surface plasmon polaritons of diffraction grating/graphene interface in which arise the interference between the vortex period caused the magnetic field and the period of diffraction metal grating. In the article [25] the group-theoretical methods and first-principles calculations to analyze a set of coupled structural distortions that underlie the polar charge and orbitally ordered antiferromagnetic ground state of A-site ordered SmBaMn$_2$O$_6$ have been developed. The authors show that these distortions play a key role in establishing the ground state and stabilizing a network of domain wall vortices. The polar-active properties of ferroelectric films have been calculated in Landau-Ginsburg-Devonshire theory framework [26, 27, 28, 29, 30, 1, 31, 32, 33, 34, 35, 24] with account of polarization vector gradients and surface charges and dipole layers on the interface in the conditions of stability of spontaneous polarization. Under the electric voltage feeds on the electrode or sound of scanning electron microscope the polarization redistributions have been achieved success. Under the increasing the electric voltage the local reversal of polarizations [36, 34, 35, 24] has been achieved which one lied to the creation the intergrown domains in thin films. Under domains recording the velocity of changes of extrinsic electric field $E$ was negligibly small and without loss of accuracy one can suppose $\text{rot} E \approx 0$. In comparison with the tight-binding method the one-component approximation of LGD theory for polarization and displacement allows the calculation only the lowest optic and lowest acoustic soft phonon modes in ferroics. Hence the one-component approximation of LGD theory for polarization and displacement can study quantitatively the soft phonon dispersion in paraelectric SrTiO$_3$. However the one-component approximation of LGD theory for polarization and displacement cannot calculate the soft phonon dispersion in multiaxial ferroelectric, perovskites BaTiO$_3$ or (Pb,Zr) TiO$_3$ for which the tight-binding method has been applied. When the propagate waves transport high energy then the medium properties can be transformed. The motion equations were non-linear equations for specified of the creation of domain walls for data storage based on polar-active ferroelectric nanofilm with strong optical activity effects. Under the spontaneous polarization switching in multiferroic as well as ferroelectric the single domain wall as well as the controlled elements of memory of nanodomain structures with increasing density of nanodomains as well as complicate domain recording of information [36, 34, 35, 24]. It is known [37] vibrations with sine-type modulations can be induced by the impulsive stimulated Raman-scattering process resulting in sine-type oscillations. It explains the coherent oscillations observed in transparent compounds under pumping with a photon energy smaller than an optical gap [37]. In an opaque materials the abrupt modification of pump photons can trigger displaced motions of ions towards new coordinates in the excited state resulting in cosine-type oscillations [37]. The problem of the structure stability of GaN is seldom considered in the literature [38]. This is due to the nontypical properties of this compound i.e. short bond length (atomic volume equals 11.4 Å) and relatively high ionicity [38]. In the Philips scale the ionicity of GaN is 0.5 or 0.43 [38] and that makes GaN the most ionic from all the III-V compounds. Almost all discussions of phase transitions of tetrahedrally coordinated compounds consider only the β-sin and NaCl structures as possible high-pressure phases [38]. The calculations of the total energy of various structures (NaCl, NiAs, β-Sn, CsCl) compared with wurtzite show that there should be a phase transition to the rocksalt structure around 55 GPa [38]. It should be noted that the NiAs phase is very close to the NaCl by stability of domains so both phases are possible [38]. A phase transition possibly to a NaCl structure has been found around 48 GPa at increasing pressure and between 30 and 20 GPa at the downstroke [38]. The growth conditions of GaN were selected for a stable growth on substrates oriented towards the c-direction [39]. Numerous hills in the form of cones and bubbles were clearly visible on the surface [39]. There were areas with different shades of blue and gray and even had a yellow area with respect of substrates oriented of growth [39]. The physical properties of wide bandgap group-III quantum well systems are under investigation due to their application to light emitters and semiconductor lasers in the ultraviolet, blue, and green wavelength regions. Ultraviolet light-emitting diodes and lasers have recently obtained considerations due to applications to the compact biological detection systems, analytical devices, and medical diagnostics. A number of light-emitting diodes and laser diodes have been demonstrated [40, 41]. However, these structures are in the developmental stage, and there are many questions with respect to the performance and device configurations. Realizing the deep-ultraviolet semiconductor-based light-emitting diodes provides light sources for various applications,
for instance to the biological detection and the data storage. Although such devices basically need a Al$_{x}$Ga$_{1-x}$N-based quantum well with high Al contents, their fundamental optical properties remain under discussion [42, 43, 23, 44, 27, 45, 46, 47, 48, 49, 50, 21, 22, 51]. It has been proved experimentally that the surface emission from [0001]-oriented Al$_{x}$Ga$_{1-x}$N is quite weak because of the predominant optical polarization along the [0001] c direction [52, 53, 54].

The explanation of these effects may be found from the difference of structures of the valence bands in AlN and in GaN. In wurtzite GaN or AlN, the degeneracy of the p-like states at the Γ point is lifted by both crystal-field splitting and spin-orbit splitting leading to forming three valence bands at the Brillouin zone center.

Since AlN has a negative crystal field splitting energy, while GaN has a positive one, these splittings lead to the ordering of the valence band in AlN: Γ$_{7}$, Γ$_{9}$, and Γ$_{6}$. Whereas we have Γ$_{6}$, Γ$_{7}$, and Γ$_{9}$ in GaN [55, 56]. Therefore, the topmost of the valence band in AlN has the crystal field split off holes with p$_{y}$-states, while the topmost in GaN has the heavy holes with p$_{x}$-like and p$_{y}$-like states, where the axis z is directed along the hexagonal axis.

Therefore, the emission from Al$_{x}$Ga$_{1-x}$N with high (low) Al-content is polarized along (perpendicular to) the c axis. Recently, many studies have been focused on the potential application of nanostuctures, such as photonic crystal structures, nanoholes, nanodots, and nanorods [57, 58, 59, 60, 61, 62, 37, 63, 20, 64, 65, 66, 67, 15, 68, 69, 70, 29, 71, 72, 73, 48, 74]. In the studies of the technology involving the photonic band gap, it seems that, in the case of dielectric rod nanoarrays or nanocolumns [50, 21, 75, 76, 77, 78, 79, 49], a large gap is opened for the TM mode, but not for the TE one [56, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 28]. Thus, with this type of structures for laser applications, the light emission in the TM mode is obtained.

In the c-plane of InGaN/GaN quantum well systems, the compressive strain is induced in the active layer, and the light is TE-polarized [75, 96, 97, 98]. Furthermore, there is a strong internal electric field caused by the spontaneous and piezoelectric polarization charges at the interfaces of the c-plane of the InGaN/GaN quantum well. This phenomenon leads to the quantum confined Stark effect, decreases the internal quantum efficiency, and leads to the emission spectrum which is red-shifted [99, 51, 22, 100, 101, 102, 103].

In some studies [104, 77, 26, 76, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114] of interface polarization charges, alloy materials were used to make a better performance. Many works have focused on the nonpolar and semipolar planes [115, 116, 26, 98, 97]. These results have testified that the light emission will be polarized, and the quantum confined Stark effect will be reduced. However, due to a higher cost of the a- and m-plane substrates, it would be better to use the c-plane substrate. In work [108, 109, 117, 118], the c-plane of the InGaN/AlGaN quantum well structure was considered instead of that of InGaN/GaN in order to obtain a tensile strain in the quantum well layer. The previous studies and calculations have shown that the [Z]-like state is generated in nitride materials, if the quantum well layer is under a tensile biaxial strain.

Besides the nitride-based devices, the group-II oxides have been considered for highly efficient laser diodes [117, 118] and high-performance field-effect transistors [119, 120]. The induced piezoelectric field plays a significant role for both band structure and optical gain [121, 122, 123]. However, the orientation of a crystal structure significantly modifies the band structure through the strain effect [124]. It has been proved experimentally that the growth along crystal directions different from the [0001] direction leads to an increase in the quantum efficiency by decreasing the strain-induced electric field in the quantum well region, possibly leading to the ways of obtaining highly efficient white laser diodes [125]. There are the theoretical works studying the effects of crystal orientation on the piezoelectric field in a strained wurtzite quantum well [123]. However, the piezoelectric effect consists not only of a strain-induced polarization; it also takes the response of both electric field and polarization on the strain into consideration. These effects were studied in paper [123].

A deeper understanding of the influence of band structures on optical properties should help one to answer many questions. In addition, the interesting effects of strong electron-hole Coulomb interaction are presented in these materials. Many-body interactions lead to effects, which consist the screening, dephasing, bandgap renormalization, and phase-space filling [126, 127, 128, 129].

A general phenomenon of Coulomb enhancement may be explained as follows. Due to the Coulomb attraction, an electron and a hole have a larger tendency to be located near each other, than that in the case of noninteracting particles. This increase of the interaction duration leads to an increase of the optical transition probability.

In the article the presented phenomena arising from the interplay between the polarity and electronic structure include two dimensional electron gases at the interface of band (LaAlO$_3$/SrTiO$_3$) or band or Mott insulators and polarization controlled electron tunneling across ferroelectric-semiconductor or half-metal interfaces (PbTiO$_3$/(La,Sr)MnO$_3$, BiTiO$_3$/SrRuO$_3$) [29, 30, 35, 34, 130]. The analytical calculations of polar active properties (including local polarization reversal) in the proper and incipient ferroelectric-dielectric thin films within Landau-Ginsburg-Devonshire (LGD) phenomenological theory with consideration of polarization gradient and intrinsic surface energy, interface dipoles, and free charges were presented in the works [29, 30, 35, 34, 130]. Hence the latter works provides a framework to link the mesoscopic LGD semiconductor theory to the first principle calculations that can reveal the details of the electrostatic field structure at the interface.

The optical activity in Rb$_2$ZnBr$_4$, Rb$_2$ZnCl$_4$, K$_2$SeO$_4$, (NH$_4$)$_2$BeF$_4$, (N(CH$_3$)$_4$)$_2$MnCl$_4$ crystals has been measured along three directions in a temperature interval from 400K to 50K [57]. In the incommensurate phase already a nonvanishing element of the gyration tensor has been observed despite the fact that the the average crystal structure has inversion
symmetry [57]. In order to explain this phenomenological space dependent dielectric and gyration tensors being invariant with respect to the superspace group of Rb$_2$ZnBr$_4$, Rb$_2$ZnCl$_4$, K$_2$SeO$_4$, (NH$_4$)$_2$BeF$_4$, (NH(CH$_3$)$_2$)$_2$MnCl$_4$ have been considered [57, 71, 60, 61, 72, 69, 70, 29].

Recently one has seen a growing interest in systems like modulated crystals with charge or spin density waves which can be considered as crystals with a distortion which is periodic in space or in space time [71, 131, 132, 133, 134, 135, 136, 137, 57, 72]. The Euclidean symmetry of these systems was not a three-dimensional space group but a four-dimensional superspace groups [71, 57, 60, 61, 138, 139, 140, 72, 69, 70, 141, 142]. It is known that the spin-orbit coupling originates from gradients of the Coulomb potentials in the atomic cores [143, 144, 145, 146]. The spin-orbit coupling have been presented by $\Delta = \frac{\hbar}{2m_c} (\psi_x [(\nabla V \times \mathbf{p})_z] \psi_y)$, where $V$ is the microscopic crystal potential of graphene. In graphene we would have $\Delta = 0$ if the basis functions $|\psi_x\rangle$ and $|\psi_y\rangle$ were made up of basis of pure $\pi$ or $(p_z)$ orbitals. However spin-orbit coupling induces a mixing of the $\pi$ or $(p_z)$ and $\sigma$ or $(p_x,p_y)$ orbitals [147] in graphene that contributes to $\Delta$ in second order of spin-orbit coupling. In our case [73, 27, 48, 74, 148, 75, 76, 59, 77, 78, 149, 79] the strain mediates a coupling between intrinsic-spin and orbital dynamics and the lowest order contribution $p_{21}^{55} (\varepsilon_x + \varepsilon_y) s_z \sigma_z$ constitutes a renormalization of the intrinsic spin-orbit coupling $p_{21}^{55} s_z \sigma_z$. In the article [58, 63, 150, 20, 64, 151, 152, 116, 115, 65, 66, 67, 15, 68] a creation of giant spin-orbit splitting ($\sim 100$ meV) of the graphene Dirac cone up to the Fermi energy are shown to be related with Au intercalation at the graphene-Ni interface. Photoelectron spectroscopy [153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 17, 164, 165, 166] reveals the hybridization with Au 5d states as the source for this giant splitting. A sharp graphene-Au interface at the equilibrium distance accounts for only $\sim 10$ meV spin-orbit splitting and enhancement is due to the Au atoms in the hollow position that get closer to graphene and do not break the sublattice symmetry [125, 124, 123, 167, 168, 169, 170, 171, 172, 122, 14, 18, 173, 121, 120, 119, 55, 154, 128, 127, 126].

### 3 Building a free-energy functional from atomically resolved imaging: Atomic-scale phenomena in Os-doped RaFeO$_3$ with magnetic field

We have described the four-sublattice model (FSM) in accordance with experimental results [13]. The conventional LGD free-energy density is a sum of Landau, gradient, and surface energies [13]:

$$G = \int_V (G_{\text{Landau}} + G_{\text{grad}}) + G_S. \quad (1)$$

Landau free-energy expansion containing the quadratic and bilinear contributions of the A-cations displacements $A_i$ ($i = 1 \text{--} 4$) in ABO$_3$ perovskite with m3m parent phase is [13]

$$G_{\text{Landau}} = \frac{2}{3} A_i A_i + \mu (A_1 A_2 + A_2 A_3 + A_3 A_4 + A_4 A_1) + \eta (A_1 A_3 + A_2 A_4) + \frac{\beta}{4} A_i^2 A_i^2 + \sum \frac{\varepsilon_{ij}}{2} (A_i A_j^2 + A_j A_i^2) + \sum \frac{\delta_{ij}}{2} (A_i A_j^2 + A_j A_i^2) + \sum \frac{\lambda}{2} A_i A_i A_i A_i + \ldots \quad (2)$$

Here we assume that only the first term in Eq. (2) has a temperature-dependent coefficient $\alpha = \alpha_T (T - T_C)$ and all constants can depend on the global or local content of impurity (e.g. Os atoms) [13].

The gradient energy $G_{\text{grad}} = g_{ijkl} \frac{\delta A_i}{\delta x_j} \frac{\delta A_i}{\delta x_k}$ will be considered in the simplest isotropic approximation for the gradient coefficient tensor $g_{ijkl}$ and that all physical quantities are $x$ dependent [13]:

$$G_{\text{grad}} = g_{ijkl} \frac{\delta A_i}{\delta x_j} \frac{\delta A_i}{\delta x_k} = g \left( \frac{\partial A_i}{\partial x} \right)^2. \quad (3)$$

The surface energy is assumed to be a positively defined quadratic form [13]

$$G_S = \frac{\phi_s}{2} (A_1^2 + A_2^2 + A_3^2 + A_4^2) \quad (4)$$

Under research of Dzyaloshinsky substitution [13]

$$B_1 = \frac{A_1 + h_2 A_2 + h_3 A_3 + h_4 A_4}{2}, \quad B_2 = \frac{A_1 - h_2 A_2 - h_3 A_3 - h_4 A_4}{2}, \quad B_3 = \frac{A_1 - h_2 A_2 + h_3 A_3 - h_4 A_4}{2}, \quad B_4 = \frac{A_1 + h_2 A_2 - h_3 A_3 + h_4 A_4}{2},$$

and making elementary algebraic transformation listed in Ref. [13] one could rewrite Eq. (2) as follows:

$$G_{\text{Landau}} = \frac{\alpha^*}{2} B_1^2 + \frac{\beta^*}{2} (B_2^2 + B_3^2) + \gamma^* (B_4^2 + B_2^2) + \delta^* (B_2^2 + B_3^2 + B_3^2 + B_4^2) + \lambda^* B_2 B_3 B_4.$$  

The expansion coefficients are [13]

$$\alpha^* = \alpha + 2 \mu + \eta, \quad \mu^* = \alpha - \eta, \quad \eta^* = \alpha - 2 \mu + \eta, \quad \beta^* = \frac{\beta}{4} + \frac{\delta}{2}, \quad \gamma^* = \frac{\gamma}{8} - \frac{\delta}{8} - \frac{\lambda}{4}, \quad \delta^* = \frac{\delta}{2} - \frac{\lambda}{2} - \frac{\gamma}{2}, \quad \lambda^* = \frac{\lambda}{2} - \gamma - \delta + \frac{\lambda}{2}. \quad (7)$$
Hence the analysis often relies on the postulated form of macroscopic Landau-Ginsburg energy for the ferroic long-range order parameter and some predefined relationship between experimentally determined atomic coordinates and the order-parameter field [13]. Hence the R and O phases coexistence (that is observed by STEM) can be realized in the case of their energies equality. The coexistence condition \( G_R = G_O \) gives \( \alpha - \eta = \alpha + 2\mu + \eta \) i.e. \( \eta = \mu \) and the phase stability conditions are \( \alpha + 2\mu + \eta < 0 \), \( \alpha - \eta < 0 \) and \( \beta + 2\gamma + \delta + \lambda > 0 \) [13]. In the case of the weak deviations from the phase equilibrium \( \eta = \mu \), i.e., when the condition \( \eta + \mu + \zeta = 0 \) takes place along with the inequality \( |\zeta| < \mu \), one could write the free energy (6) in the following dimensionless form [13]:

\[
G_{12} \approx - (\alpha + \mu) B^2 \int [-(1 - c) b^2_t + (1 - c) b^2_{ih} + b^2_{ih} + b^2_{i2h} + \frac{1}{2} (b^2_{i1h} + b^2_{i2h}^2)] + \frac{3}{2} \mu b_0 B_0 k \mathcal{H}_z \sigma_z, 
\]

(8)

where the order parameters \( B_i = B_s b_i \) \((i = 1, 2)\), the spontaneous value \( B_S = 2 \sqrt{-\frac{e+2m+\eta}{\beta+2\gamma+\delta+\lambda}} \), dimensionless coupling constant \( \chi = \frac{\chi}{\beta} - \frac{\beta}{\beta+2\gamma+\delta+\lambda} \), the gradient coefficient \( h = B^2 \), and the parameter \( c = \frac{\chi}{\alpha+\mu} \) is the sublattice asymmetry constant [13]. To study the conditions between coexisting R and O phases we have solved analytically the Euler-Lagrange equations with consideration of the Landau levels by variation of the energy (8). Hence we have found the following the Euler-Lagrange equations

\[
(-1 - c) + b^2_{i1h} + b^2_{i2h} + \chi b^2_{i2h} + b_{i1h} = 0, \quad (-1 - c) + b^2_{i1h} + b^2_{i2h} + \chi b^2_{i2h} + b_{i1h} = 0,
\]

(9)

from which we have specified the \( b_{i1h} \) variable via \( b_{i2h} \) variable

\[
b^2_{i2h} = (1 - c) - b^2_{i1h} + \chi b^2_{i1h},
\]

(10)

Hence we have found the following equation

\[
\left( 1 - \chi^2 \right) b^2_{i1h} \Psi + (1 - \chi) b_{i1h} \Psi = (1 - c) \Psi,
\]

(11)

where \( \Psi = \{ b_{i1h}, b_{i2h} \}. \) The covariant derivative \( \nabla = \partial + (i e/hc)A \) includes the vector potential in the symmetric gauge \( A^{\text{ext}} = (\mathcal{H}_z y, \mathcal{H}_y z) \) corresponding to the external magnetic field applied perpendicular to the plane along the positive \( z \) axis

\[
\left( 1 - \chi^2 \right) b^2_{i1h} \Psi + (1 - \chi) k_{i1h} + \frac{\mathcal{H}_z}{hc} y^2 \Psi = (1 - c) \Psi,
\]

(12)

in which introducing \( y^* = y + y_0 \) where \( y_0 = s^2 k_x, s^2 = \frac{h e}{\mathcal{H}_y} \) we have found

\[
\left( 1 - \chi^2 \right) b^2_{i1h} \Psi + (1 - \chi) h s^{-1}(s^{-1} y^* - \xi^2) \Psi = (1 - c) \Psi,
\]

(13)

as well as

\[
\left( 1 - \chi^2 \right) b^2_{i1h} \Psi + (1 - \chi) h s^{-1} (\xi^2 - \xi^2) \Psi = (1 - c) \Psi,
\]

(14)

where \( \xi = s^{-1} y^* \). We seek the solution of Eq. (15) in the form \( \Psi = \sum_n C_n \exp^{-\xi^2/2 H_n(\xi)} \)

\[
(1 - \chi^2) b^2_{i1h} \Psi + (1 - \chi) h s^{-1} (\xi^2 - \xi^2) \Psi = (1 - c) \Psi,
\]

(15)

after the substituting solution in equation Eq. (15) and by premultiplication on seeking distribution series of Chebyshov-Hermite functions and by integrated at defined space we have found

\[
\sum_n \int_{-\infty}^{\infty} d\xi \exp^{-\xi^2/2 H_m(\xi)} (1 - \chi^2) b^2_{i1h} C_n \exp^{-\xi^2/2 H_m(\xi)} + \int_{-\infty}^{\infty} d\xi \exp^{-\xi^2/2 H_m(\xi)} (1 - \chi) h s^{-1} (\xi^2 - \xi^2) C_n \exp^{-\xi^2/2 H_m(\xi)} =
\]

\[
= \sum_n \int_{-\infty}^{\infty} d\xi \exp^{-\xi^2/2 H_m(\xi)} (1 - c) C_n \exp^{-\xi^2/2 H_m(\xi)} ,
\]

(16)

with taking into account the condition of normalization of wave function and the following recurrence relations

\[
\int_{-\infty}^{\infty} d\xi \exp^{-\xi^2/2 H_m(\xi)} \exp^{-\xi^2/2 H_m(\xi)} = \frac{\sqrt{\pi} n! m!}{\sqrt{1 + \chi ont}} \delta_{n,m},
\]

\[
\exp^{-\xi^2/2} \Psi_n(\xi) = \sqrt{\frac{\pi}{2}} \Psi_{n-1} + \sqrt{\frac{\pi}{2}} \Psi_{n+1}, \quad \frac{\partial}{\partial \xi} \Psi_n(\xi) = \sqrt{\frac{\pi}{2}} \Psi_{n-1} - \sqrt{\frac{\pi}{2}} \Psi_{n+1},
\]

we have specified the following algebraic equation

\[
(1 - \chi^2) b^2_{i1h} + 2(1 - \chi) h s^{-1}(n + \frac{1}{2}) = (1 - c),
\]

(17)

from which we have found

\[
b_{i1h} = \pm \sqrt{\frac{(1 - c)(1 - \chi^2)}{(1 + \chi^2)}} - \frac{2h c^{-2}}{(1 + \chi^2)} (n + \frac{1}{2}),
\]

(18)

as well as
\[ b_{2s} = \pm \sqrt{(1 - c) - 2\hbar s^{-2}(n + \frac{1}{2}) - \chi \left( \frac{1 - c}{1 - c} - \frac{2\hbar s^{-2}}{1 + x} \right)(m + \frac{1}{2})}, \]  
(19)\]

where \( n, m = 0, 1, 2, 3, 4, \ldots \). The solution (19) can be rewritten also like

\[ b_{2s} = \pm \sqrt{(1 - c) - 2\hbar s^{-2}(n - m) - \left( \frac{1 - c}{1 - c} - \frac{2\hbar s^{-2}}{1 + x} \right)(m + \frac{1}{2})}, \]  
(20)\]

where \( s^{-2} = \frac{e^{\nu_e}}{\hbar c} \). The solution (18) can be found also like

\[ b_{1s} = \pm \sqrt{\Delta^2_{1s} + 2n\hbar |e|\nu_e|/(\hbar c(1 + \chi))}, \]  
(21)\]

where \( \Delta^2_{1s} = \frac{(1 - c)}{(1 - x)} + h|e|\nu_e|/(\hbar c(1 + \chi)) \). Hence the Hall conductivity can be found like for graphene in the form of expression [174, 175, 176]

\[ \sigma_{xy} = \frac{e^2N_{s,0}e^2}{2\pi\hbar} \left[ 1 + 2\sum_{\mu=1}^{\infty} \theta(\mu - M_0) \right] = \frac{e^2N_{s,0}e^2}{2\pi\hbar}(1 + 2[\frac{\nu^2\hbar c(1 + \chi)}{2\hbar s_{0}\nu_e^2}]), \]  
(22)\]

via which we have specified the Integer Hall conductivity for thin Os\(_{x}\)FeO\(_{y}\) nanofilms in the form

\[ \sigma_{xx}^{1s} = \frac{e^2N_{s,0}e^2}{2\pi\hbar}(1 + 2[\frac{\nu^2\hbar c(1 + \chi)}{2\hbar s_{0}\nu_e^2}]), \]  
(23)\]

The solution (20) can be found also like

\[ b_{2s} = \pm \sqrt{\Delta^2_{2s} + 2n\hbar |e|\nu_e|/(\hbar c(1 + \chi))}, \]  
(24)\]

where \( \Delta^2_{2s} = (1 - c)(1 - \frac{\chi}{(1 - x)}) + h|e|\nu_e|/(\hbar c(1 + \chi)) \). For which we have specified the Integer Hall conductivity for thin Os\(_{x}\)FeO\(_{y}\) nanofilms in the form

\[ \sigma_{xy}^{2s} = \frac{e^2N_{s,0}e^2}{2\pi\hbar}(1 + 2[\frac{\nu^2\hbar c(1 + \chi)}{2\hbar s_{0}\nu_e^2}]), \]  
(25)\]

where \( \lfloor x \rfloor \) denotes the integer part of \( x \), \( N_f = 2 \), so \( \nu_{xy} = 2n + 1 \).

4 Phase transition and quantum Hall effect in graphene

4.1 Unconventional Integer Quantum Hall Effect in Graphene

The low-energy quasiparticles excitations in graphene are conventional described in terms of a four-component Dirac spinor [174, 175, 176] \( \Psi_t^{\sigma} = (\psi_{K\pi}, \psi_{K\pi'}, \psi_{K'\pi}, \psi_{K'\pi'}) \) which combines the Bloch states with spin \( \sigma = \pm 1 \) on the two different sublattices (A,B) of the trigonal graphene band structure and with momenta near the two inequivalent points (K,K') at the opposite corners of the two-dimensional Brillouin zone. The free quasiparticle Hamiltonian can be recast in the relativisticlike form with the Fermi velocity \( \nu_F = 10^6 m/s \) like velocity of light in relativistic physics [174, 175, 176],

\[ H_0 = -i \nu_F \int d^2r \bar{\Psi}_t(y^1\hbar \nabla_x + y^3\hbar \nabla_y)\Psi_t, \]  
(26)\]

where \( \bar{\Psi}_t = \Psi_t^\dagger y^0 \) is the Dirac conjugated spinor and summation over spin \( \sigma \) is understood. In Eq. (26) \( \gamma^\nu \) with \( \nu = 0, 1, 2 \) are 4 \times 4 gamma matrices belonging to a reducible representation \( \gamma^\nu = \tilde{\tau}_2 \otimes (\tau_3, \epsilon \tau_2, -\epsilon \tau_1) \) where the Pauli matrices \( \tilde{\tau}, \tau \) act in the subspaces of the valley (K,K') and sublattices (A,B) indices, respectively. The matrices satisfy the usual anticommutation relations \( \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, g^{\mu\nu} = (1, -1, -1, 1) \), \( \mu, \nu = 0, 1, 2 \). The covariant derivative \( \nabla = \partial + (\nu e/\hbar c)A \) includes the vector potential in the symmetric gauge \( A^{ext} = (-B_y/2, B_x/2) \) corresponding to the external magnetic field applied perpendicular to the plane along the positive z axis. In the four-component spinor representation the Coulomb interaction has the form [174, 175, 176]

\[ H_{int} = \frac{\hbar \nu_F}{2} \int d^2r d^2r' \bar{\Psi}_t(r)\gamma^0\Psi_t(r') \frac{\epsilon}{|r-r'|} \bar{\Psi}_t(r') \gamma^0 \Psi_t(r), \]  
(27)\]

where the coupling \( g = e^2/\epsilon_0 \hbar c \nu_F \) and \( \epsilon_0 \) is the dielectric constant (our convention is \( e > 0 \)). The total Hamiltonian \( H_{tot} = H_0 + H_{int} \) possesses \( U(4) \) symmetry discussed in the articles [174, 175, 176]. The chemical potential is introduced through adding the term \( -\nu \Psi_0^\dagger \Psi = -\nu \Psi \bar{\Psi} \) in \( H_{tot} \) (this term preserves the \( U(4) \) symmetry). The Zeeman interaction term is included by adding the term \( \mu_B B\Psi^\dagger \sigma_2 \Psi = \Psi^\dagger \sigma_3 \Psi \), where \( \sigma_3 \) matrix acts on spin indices. Here \( \mu_B = e\hbar /2mc \) is the Bohr magneton and we took into account that the Lande factor for graphene \( g_L \approx 2 \). Let us first consider case with no spin splitting (the Zeeman term is ignored). We have derived the thermodynamic potential per unit area in a strong magnetic field \( B \), when the dynamics of the lowest Landau level (LLL) dominate [174, 175, 176].
\[ \Omega^\ast(\Delta, \mu) = \frac{1}{\pi l^2} \{ \Delta f(\Delta, \mu) - \frac{b_l}{b_l} f^2(\Delta, \mu) + 2T \times \Re e[\ln \Gamma(\frac{1}{2}(\mu+\Delta)) + \frac{1}{2} + \ln \Gamma(\frac{1}{2}\mu - \frac{\Delta}{2}) + \frac{1}{2}] - 2\ln \Gamma(\frac{1}{2}\mu + \frac{\Delta}{2}) + \frac{1}{2}\} \}, \]

where \( l = \sqrt{\hbar c/|eB|} \) is the magnetic length, \( L(B) = \sqrt{\hbar v_F^2|eB|/c} \) is the Landau scale, \( \Gamma(x) \) is the Euler gamma function, \( \gamma \) is a LLL impurity scattering rate, and the function \( f(\Delta, \mu) \) is \([174, 175, 176] \)

\[ f(\Delta, \mu) = \frac{1}{\pi} \Im m[\Psi(\frac{1}{2}(\mu+\Delta)) + \frac{1}{2} - (\Delta \rightarrow -\Delta)], \]

with the digamma function \( \Psi(x) = \frac{d}{dx} \ln \Gamma(x) \). The dimensionless parameter \( b \) in Eq. (28) reads

\[ b = \frac{\delta^2}{\gamma_0} \int_0^\infty \frac{dk \exp^{-ik}}{1+k\chi_0}, \]

where \( \chi_0 \approx 0.56\sqrt{2}\pi g \). The gap equation \( \partial \Omega^\ast/\partial \Delta = 0 \) for \( \Delta \) takes the form \([174, 175, 176] \)

\[ \Delta - bL(B)f(\Delta, \mu) = 0. \quad (31) \]

Let us show how the generation of the gap affects the form of the Hall conductivity \( \sigma_{xy}^\ast \) valid for large \( B \). For large enough \( B \) (or small enough \( \gamma \) scattering rate) it becomes a strong first order phase transition \([174, 175, 176] \)

\[ \sigma_{xy}^\ast = -\frac{2\pi^2}{h} \frac{sgn(eB)\mu}{\theta} [\{ |\mu| - \Delta(\mu, B) \}], \quad (33) \]

The lowering of the degeneracy of the LLL generating the gap in graphene is connected with the spontaneous breakdown of the initial U(4) symmetry down to U(2)_1 × U(2)_2. The expression for diagonal conductivity is found in the articles \([174, 175, 176] \)

\[ \sigma_{xx}^\ast = \frac{2\pi^2}{h\gamma^2} \frac{\gamma^2 + \Delta^2}{}, \quad (34) \]

for the trigonal band structure of graphene \( E = \pm \sqrt{2h v_F^2|eB|/c + \Delta^2} \). Hence the thermodynamic potential \( \Omega \), the Hall conductivity \( \sigma_{xy}^\ast \) and the diagonal conductivity \( \sigma_{xx}^\ast \) are now \([174, 175, 176] \)

\[ \Omega(\Delta, \mu) = \frac{1}{2} \{ \Omega^\ast(\Delta, \mu_+) + \Omega^\ast(\Delta, \mu_-) \}, \]

\[ \sigma_{xy} = \frac{1}{2} [\sigma^\ast_{xy}(\mu_+) + \sigma^\ast_{xy}(\mu_-)], \]

\[ \sigma_{xx} = \frac{1}{2} [\sigma^\ast_{xx}(\mu_+) + \sigma^\ast_{xx}(\mu_-)], \]

where \( \mu_\pm = \mu \pm \delta_\mu \) with \( \delta_\mu = \mu_B B \) being the Zeeman term. This term breaks explicitly the U(4) symmetry down to U(2)_1 × U(2)_2 at phase transitions driven by magnetic field. The latter is obtained by a simple compilation of the experimentally observed transition points between the corresponding plateaus in Hall conductivity. The best fit that was found is \([174, 175, 176] \)

\[ \mu = 0.5(V_g - V_0) + 7.0\mu_B(V_g - V_0) \sqrt{|V_g - V_0|}. \quad (38) \]

### 4.2 Unusual quantization of the Hall conductivity in graphene

Hence the expression for Hall conductivity we have found like

\[ \sigma_{xy} = -\frac{e^2N_s N_g|eB|}{4\pi} [\tanh \frac{\mu+\Delta}{2T} + \tanh \frac{\mu-\Delta}{2T} + 2 \sum_{n=1}^\infty (\tanh \frac{\mu+M_n}{2T} + \tanh \frac{\mu-M_n}{2T})], \quad (39) \]

where \( \tanh \frac{\omega_B - \mu}{2T} = 1 - 2n_F(\omega) \) so we have rewritten Eq. (39) via Fermi distribution

\[ \sigma_{xy} = -\frac{e^2N_s N_g|eB|}{2\pi} \sum_{n=1}^\infty (2n + 1) [n_F(M_n) + n_F(-M_n) - n_F(M_{n+1}) - n_F(-M_{n+1})], \quad (40) \]

where \( M_n = \pm \sqrt{2n\hbar v_F^2|eB|/c + \Delta^2} \). Now we rewrite Eq. (39) as follows

\[ \sigma_{xy} = -\frac{e^2N_s N_g|eB| \mu}{2\pi \hbar} V_g, \quad (41) \]
with the filling factor
\[ \text{sgn } \mu y_B = \frac{1}{2} \left( \tanh \frac{\mu + \Delta}{2T} + \tanh \frac{\mu - \Delta}{2T} + 2 \sum_{n=1}^{\infty} \left( \tanh \frac{\mu + M_n}{2T} + \tanh \frac{\mu - M_n}{2T} \right) \right), \]
(42)
where \( h = 2\pi \hbar \), and using that \( \text{sgn}(\omega/2T) = \text{sgn}(\omega) \) at \( T \to 0 \) we obtain from Eq. (44)
\[ \sigma_{xy} = -\frac{e^2 N_s \text{sgn}(\epsilon_B)}{2\pi n} \left[ 1 + 2 \sum_{n=1}^{\infty} \frac{\text{sgn}(\mu - M_n)}{\theta(\mu - M_n)} \right] = -\frac{e^2 N_s \text{sgn}(\epsilon_B)}{h} \left( 1 + 2 \left[ \frac{\mu^2}{2h|\mu|}\psi \right] \right), \]
(43)
where \([x]\) denotes the integer part of \( x \), \( N_f = 2 \), \( \nu_B = 2n + 1 \).

### 4.3 Calculation of the conductivity

In the bare-bubble approximation the expression for the diagonal conductivity in the limit of \( B \to \infty \) can be obtained from Eqs. (3.11), (3.12) in the second paper in Ref. [175]
\[ \sigma^*_{xx} = \frac{e^2}{\pi^2} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2}, \]
(44)
the integrals in this expression can be evaluated exactly as follows
\[ I = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} = \pi i \int_{-\infty}^{\infty} d\omega \text{exp}^{i(\omega - \Delta)} = 2T \pi i \int_{-\infty}^{\infty} d\omega \text{exp}^{-i[\gamma_{tr} - (\omega - \Delta)]} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2}, \]
(45)
The integral over \( x \) is evaluated by means of the formula from Ref. [174]. Then we get
\[ I = 4\pi T \int_{0}^{\infty} \frac{d\omega}{2\pi} \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} = \frac{2}{\pi} \pi i \int_{-\infty}^{\infty} d\omega \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right). \]
(46)
Thus we obtain
\[ \sigma^*_{xx} = \frac{e^2}{\pi^2} \rho \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2}, \]
(47)
One can check that for \( \mu = 0 \) and \( T \to 0 \) Eq. (48) reduces to Eq. (34). Now we derive the expression for the dc Hall conductivity. In the limit \( B \to \infty \), Eqs. (3.14), (3.15) in the second paper in Ref. [175] yield
\[ \sigma^*_{xy} = -\frac{e^2 \text{sgn}(\epsilon_B)}{4\pi^2 T} \int_{-\infty}^{\infty} d\omega \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} \frac{2}{\pi} \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) + \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right), \]
(48)
We first consider the terms with \text{arctan} functions taking derivative with respect to \( \Delta \):
\[ \frac{\partial \sigma_{xy}^{(i)}}{\partial \Delta} = \frac{e^2 \text{sgn}(\epsilon_B)}{4\pi^2 T} \int_{-\infty}^{\infty} d\omega \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} \left[ \frac{\gamma_{tr}}{\gamma_{tr} + (\omega - \Delta)^2} - \frac{\gamma_{tr}}{\gamma_{tr} + (\omega - \Delta)^2} \right], \]
(50)
Then we use Eq. (47) and find
\[ \frac{\partial \sigma_{xy}^{(i)}}{\partial \Delta} = -\frac{e^2 \text{sgn}(\epsilon_B)}{2\pi^2 T} \rho \left[ \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) - \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) \right]. \]
(51)
Therefore
\[ \sigma_{xy}^{(i)} = -\frac{e^2 \text{sgn}(\epsilon_B)}{\pi} \pi \left[ \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) + \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) \right], \]
(52)
Here we took into account the fact that because of the condition \( \sigma_{xy}^{(i)}(\Delta = \infty) = 0 \) (see Eq. (49)) and the known asymptotics of the \( \Psi \) function the integration constant equals zero. As to the integrals of the two first terms in square brackets in Eq. (49) they are calculated by means of the formula
\[ \int_{-\infty}^{\infty} d\omega \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} = \frac{2}{\pi} \pi \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right). \]
(53)
Its derivation is as follows:
\[ \int_{-\infty}^{\infty} d\omega \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} = \pi i \int_{-\infty}^{\infty} d\omega \frac{1}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2} = \pi i \int_{-\infty}^{\infty} d\omega \text{exp}^{i(\omega - \Delta)} = 2T \pi i \int_{-\infty}^{\infty} d\omega \text{exp}^{-i[\gamma_{tr} - (\omega - \Delta)]} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\gamma_{tr}}{\cosh \left( \frac{\gamma_{tr} \omega}{2T} \right) + \gamma_{tr} + (\omega - \Delta)^2}, \]
(54)
\[ = 4\pi T^2 \int_{0}^{\infty} d\omega \text{sin} \left( \frac{\omega - \Delta}{2T} \right) \text{exp}^{i[\gamma_{tr} - (\omega - \Delta)]} \frac{d\omega}{2\pi} \text{exp}^{i[\gamma_{tr} - (\omega - \Delta)]} = -\frac{e^2 \text{sgn}(\epsilon_B)}{2\pi^2 T} \rho \left[ \Psi' \left( \frac{\gamma_{tr} + (\omega - \Delta)}{2T} + \frac{1}{2} \right) \right]. \]
Combining this contribution with that in Eq. (52) we arrive at Eq. (32).

The free energy expansion in powers of order parameters is employed in Landau-Ginsburg-Devonshire (LGD) free energy and can be used in the phase-field modeling of macro and nanosized ferroelectrics \cite{13}. Understanding of ferroic behavior at surfaces, interfaces, and defects as well as the nature of ferroelectric states considerably advanced in the last decades with the advancement of scanning transmission electron microscopy (STEM) \cite{13}. Probing the unit-cell level symmetry breaking via STEM allowed the determination of direct atomic positions, from which the spatial distributions of order parameters fields can be mapped \cite{13}. Hence here we derive a model LGD-type free energy describing directly observable degrees of freedom available from atomic-resolution STEM \cite{13}. We have proposed a theoretical four-sublattice model (FSM) for analytical description of cation displacement in (anti)ferroelectric-antiferrodistortive perovskites of ABO$_3$ type, that explain the coexistence of rhombohedral (R), orthorhombic (O), and spatially modulated (SM) phases observed by atomic-resolution STEM \cite{13}. Using this approach we atomically resolve and theoretically model the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in perovskites of Os$_{x}$Ra$_{1-x}$FeO$_3$ polymorphs with magnetic field. Hence for over 50 years of researches of ferroic materials the properties of these materials were explored using the combination of scattering techniques that provides the information on the nature and symmetry of order parameters and macroscopic measurements that provided the information on the corresponding expansion coefficients and the nature of phase transitions \cite{13}.

In a landmark paper in 1982, Thouless, Kohmoto, Nightingale, and den Nijs \cite{15} analyzed the uniform-field Hall effect in a strong periodic potential that was known to lead to an intricate spectrum, the so-called Hofstadter butterfly (see Fig. 1); they showed that it gives rise to an integer QHE under certain conditions, i.e., whenever the chemical potential lies in a gap \cite{14}. Indeed, the Hall conductance was shown to map to a topological invariant associated with filled bands - the (first) Chern number. In this model, time-reversal symmetry is broken by a spatially inhomogeneous magnetic field with zero average, and the Hall conductance again equals the Chern number of the band \cite{14}. Six years later in another striking development Haldane answered the second question showing by an explicit construction of a tight-binding model on a honeycomb lattice that a quantized Hall conductance can arise from a fully filled band even in the absence of a net magnetic field \cite{14}.

A Landau level involves a set of exactly degenerate single-particle states and thus, at a fractional filling, the kinetic energy alone does not select a ground state, but instead, it falls to the interactions to force the issue \cite{14}. By contrast, a Chern band typically will have a significant dispersion that will select a unique kinetic-energy-dominated ground state at reasonable interaction strengths, as it does in all metals. Recognizing this, all three paper \cite{16,17,18} devote considerable effort to constructing lattice models with nearly flat (degenerate) Chern bands. The authors of article \cite{17} construct a flattened version of Haldane's model on a square lattice. They note that while a fully flattened model requires the inclusion of electron hopping over arbitrarily large distances, the hopping amplitudes decrease exponentially, which allows a relatively flat band to be constructed by keeping a small set of hopping amplitudes.

Figure 1. (Color online) A colored Hofstadter butterfly (Ref. [14]): This figure represents the phase diagram of Bloch electrons in a uniform magnetic field. The horizontal axis indicates the chemical potential and the vertical axis the flux through the system. Each color corresponds to a distinct structural phase transition with a particular quantized value of the Hall conductance.
relevant flatness parameter, which should be large for the effects of interactions to be important, is the ratio of the band gap (which sets a bound on the strength of the interactions one can safely include) to the bandwidth and they show how to get this number up to seven with just second-neighbor interactions. Similarly the authors of articles [16, 18] construct models on the kagome and checker-board lattices, which also exhibit large values of the flatness parameters [14]. With a flat Chern band in hand, the authors of the paper [17] introduce interactions and study the system at a fractional filling of 1/3 through numerical computation on a sized system. They find two of the classic signatures of the 1/3 FQHE state: a fractional quantum Hall conductance that was close to the filling fraction, and a nontrivial ground-state degeneracy with periodic boundary conditions. As a test, they vary the band structure continuously to a topologically trivial band and find that these features go away. In a related piece of unpublished work, another group finds similar results at filling of 1/3 and 1/5 [14]. Altogether, this work offers strong evidence that fractionally filled Chern bands do indeed exhibit the FQHE [14].

In this earlier work, the authors [14] studied a fixed filling factor while varying the flux per plaquette from small values and large unit cells, where the standard Landau level description holds, to somewhat larger flux values and smaller unit cells, where that description broke down. As they were able to change this parameter without any evidence of encountering a phase transition, the latter limit constituted an observation of the FQHE in the presence of strong lattice effects [14].

5 Incommensurately modulated sinusoidal polarization waves $Q_T$, $Q_P$, $Q_b$ in stationary Schrödinger boundary problem for domain wall for data storage of SmBaMn$_2$O$_6$

nanosize ferroics with magnetic field

We have found the following Landau-free energy expansion for specify domain walls for data storage of thin SmBaMn$_2$O$_6$ as well as PmBaMn$_2$O$_6$ nanofilms with the P4mm space group of reference data [25, 177]

$$F_i = \frac{1}{2} \alpha_i Q_i^2 + \frac{1}{4} \beta_i Q_i^4 + \frac{1}{2} \alpha_i (s_1^2 + s_2^2) + \frac{1}{2} \beta_i (s_1^2 + s_2^2)^2 + \frac{1}{2} \gamma s_1^2 s_2^2 + \frac{1}{2} \alpha_b Q_b^2 + \frac{1}{2} \alpha_b Q_b^2 + F_{tss} + F_{bs} + F_{tbp},$$

where $F_{tss} = \delta_{tss} Q_T s_1 s_2$, $F_{bs} = \delta_{bs} Q_b (s_1^2 - s_2^2)$, as well as $F_{tbp} = \delta_{tbp} Q_T Q_b Q_P$.

The Euler-Lagrange dynamic equations for the order parameter component $Q_T(k_t, z)$ further denoted as $f_0(k_t, z) \equiv \phi^{(ij)}(z, k_t)$ we have specified from the Langrange function of ferroic film by varying of variables of the Functional Eq. (55)

$$\begin{cases} 
\alpha_t Q_T + \beta_t Q_T^3 + \delta_{tss} s_1 s_2 + \delta_{tbp} Q_b Q_P = 0, \\
\alpha_b Q_b + \delta_{bs} (s_1^2 - s_2^2) + \delta_{tbp} Q_T Q_P = 0, \\
\alpha_b Q_P + \delta_{tbp} Q_T Q_b = 0,
\end{cases}$$

where $s_1^2 + s_2^2 = Q_T^2$, $s_1 = 0.88 Q_t$, $s_2 = 0.47 Q_t$, $Q_T^2 + Q_P^2 + Q_b^2 = 1$.

From equations system Eqs. (56) one can find

$$\alpha_b Q_P + \delta_{tbp} Q_T Q_b = 0,$$

the order parameter $Q_b$

$$Q_b = -\frac{\alpha_b Q_P}{\delta_{tss} Q_T},$$

and substituting $Q_b$ in Eqs. (56) we have derived

$$-\frac{\alpha_b \alpha_b Q_P}{\delta_{tss} Q_T} + \delta_{tbp} Q_T Q_P + \delta_{bs} (s_1^2 - s_2^2) = 0,$$

as well as

$$-\alpha_b \alpha_b Q_P + \delta_{tbp} Q_T Q_P + \delta_{bs} (s_1^2 - s_2^2) \delta_{tbp} Q_T = 0,$$

and after elementary transformations one can find

$$Q_P (-\alpha_b \alpha_b + \delta_{tbp} Q_T^2) = \delta_{bs} (s_1^2 - s_2^2) \delta_{tbp} Q_T.$$

Hence we have found

$$Q_P = \frac{\delta_{tss} (s_1^2 - s_2^2) \delta_{tss} Q_T}{(-\alpha_b \alpha_b + \delta_{tss} Q_T^2)},$$

as well as
\[ Q_b = -\alpha_p \frac{\delta_{\alpha a}(z^2 - \gamma^2)}{(-\alpha_p z + \delta_{\alpha a}^T z^2)}. \]  

Substituting Eqs. (62), (63) into Eqs. (56) we have specified the equation for \( Q_T \) order parameter as follows

\[ \alpha_i Q_T + \beta_i Q^2_T + \delta_{is} s_i s_2 - \frac{\alpha_p \delta^2_{\alpha a}(z^2 - \gamma^2)}{(-\alpha_p z + \delta_{\alpha a}^T z^2)^2} Q_T = 0, \]

after elementary transformation we have obtained

\[ (\alpha_i Q_T + \beta_i Q^2_T + \delta_{is} s_i s_2)(-\alpha_p \alpha_b + \delta_{is}^2 Q^2_T) - \alpha_p \delta^2_{bs} (s_1^2 - s_2^2) \delta^2_{\alpha b} Q_T = 0, \]

as well as

\[ (\alpha_i Q_T + \beta_i Q^2_T + \delta_{is} s_i s_2)(\alpha_i^2 \alpha_b^2 - 2\alpha_p \alpha_b \delta^2_{\alpha b} Q^2_T + \delta^4_{\alpha b} Q^4_T) - \alpha_p \delta^2_{bs} (s_1^2 - s_2^2) \delta^2_{\alpha b} Q_T = 0, \]

and hence

\[ \alpha_i \alpha^2 \alpha_b^2 Q_T - 2\alpha_i \alpha_b \delta^2_{\alpha b} Q^2_T + \alpha_i \delta^4_{\alpha b} Q^4_T + \alpha_i^2 \alpha_b^2 Q_T - 2\alpha_p \alpha_b \delta^2_{\alpha b} Q^2_T + \beta_i \delta^4_{\alpha b} Q^4_T + \delta^2_{\alpha b} \delta^2_{\alpha b} Q_T = 0. \]  

Let us known a ferroic film of thickness \( h \) confined in the \( z \) direction in the region \( s \in [-h/2..h/2] \), hence \( k_l \rightarrow -i \frac{\omega}{c} \). The film surfaces are covered by two ideally conducting soft electrodes, which do not make any mechanical state and we have specified the nanofilm by the wave function with orthonormal basis \( |1, \zeta \rangle \) like as

\[ \langle \Psi_{\nu, \zeta} | k_i \rangle = \sum_{m=1}^{\infty} \langle \psi^{(1)}_{k_i} | 1, \zeta \rangle \sum_{m=1}^{\infty} \langle \psi^{(2)}_{k_i} | 2, \zeta \rangle, \]

where \( \zeta \) is a chirality, \( \psi_{m}(Z) = \sqrt{\frac{\pi}{h}} \sin (\pi m(Z + \frac{1}{2})) = \sqrt{\frac{\pi}{h}} \sin (\pi m Z), Z = (z + \frac{1}{2}) \).

We have specified the depolarization field \( E^{d}_{(n, z)}(k, z) \) by Eq. (69) and which has been involved into Schrödinger boundary eqn. (82), (83) determines the chirality form of solution of Eqs. (82), (83).

\[ E^{d}_{(n, z)}(Q_T(k, z)) = \begin{vmatrix} \frac{-Q_T(k, z)}{E_{0}(k, z)} + \int_{0}^{z} dz' \int_{-\delta x}^{\delta x} [Q_T(k, z') \sin (K x')][\text{sin}(K(h-z'))[\text{sin}(K(k)) \text{sin}(K(z')) \text{sin}(K(h-z'))]\text{sin}(K(k)) \text{sin}(K(z')) \text{sin}(K(h-z'))] \end{vmatrix}. \]

In the simplest one-component one-dimensional case considered hereinafter the bulk part of the free energy \( F \) which depends on one-component order parameter \( Q_T \) coupled with the order parameters \( Q_s \) and \( Q_b \) which depend only on the coordinate \( x \) and their gradients has the following form

\[ F \approx \int_{0}^{h} dz \int_{-\infty}^{\infty} dx dy \left\{ \frac{1}{2} \alpha_i Q^2_T + \frac{1}{2} \beta_i Q^4_T + \frac{1}{2} \alpha_i (s_1^2 + s_2^2) + \frac{1}{2} \beta_i (s_1^2 + s_2^2)^2 + \frac{1}{2} \gamma_i (s_1^2 + s_2^2)^3 + \frac{1}{2} \gamma_i (s_1^2 + s_2^2)^2 \right\} + \frac{1}{2} \alpha_i Q^2_T + \frac{1}{2} \beta_i Q^4_T + F_{ext} + F_{bxs} + F_{ribp} - \]  

\[ Q_T \left[ \frac{E_{0}^{(1)}}{2} + E_{0}^{(2)} \right] + \frac{1}{2} \left( \frac{Q_T}{\gamma x_1} \right)^2 + \frac{1}{2} \left( \frac{Q_T}{\gamma y_1} \right)^2 + \frac{1}{2} \left( \frac{Q_T}{\gamma z_1} \right)^2 \int_{-\infty}^{\infty} dxdy\left[ Q_T^2 \left| \gamma - Q_T^2 \right| \right], \]

where

\[ Q_b = \frac{\delta_{bs}(z^2 - \gamma^2)}{(-\alpha_p z + \delta_{bs}^T z^2)}, \]

\[ Q_b = -\alpha_p \frac{\delta_{bs}(z^2 - \gamma^2)}{(-\alpha_p z + \delta_{bs}^T z^2)}, \]

as well as

\[ \alpha_i = \alpha(T) - \frac{(Q_{s_{11}} + Q_{s_{11}}) h_1}{s_{11} + s_{11}} - 2 \frac{(Q_{s_{11}} + Q_{s_{11}}) h_1 - 2s_{11}Q_{s_{11}}}{s_{11} + s_{11}} \epsilon_{0}, \beta_i = \beta + 2 \frac{(Q_{s_{11}} + Q_{s_{11}}) h_1 - 2s_{11}Q_{s_{11}}}{s_{11} + s_{11}} \epsilon_{0}, \]

\( s_{11} \) is an elastic coefficients. Hence we have specified the stationary Schrödinger boundary problem in space confined medium or planar waveguide as follows

\[ \Gamma \left[ Q_T^2 + Q_T^2 + \alpha Q_T^2 + \beta Q_T^4 + \gamma Q_T^2 + \gamma Q_T^2 - g_0 \frac{\delta_{s}^2 Q}{\delta x^2} - (g_1 + Q_T^2 \frac{\delta^2 Q}{\delta x^2}) + \right] \]

\[ \alpha' = \alpha_i \alpha_b^2 - \alpha_p \delta_{\alpha a}^2 (s_1^2 - s_2^2) \delta_{\alpha b}^2 \]
The solutions of Schrödinger equations can be found in the form Eq. (68) i.e. 

\[ \phi \]

Let us known the following integral relations for the found solutions algebra equation systems

\[ \text{and with boundary conditions} \]

\[ (a^\prime Q_T - g_3 \frac{\partial Q_T}{\partial x_z} - w_1 \frac{\partial Q_T}{\partial x_z} - v_1 Q_T \frac{\partial Q_T}{\partial x_z} (Q_T + \frac{\partial Q_T}{\partial x_z})) |_{x_z=0} = 0, \]

\[ (a^\prime Q_T + g_3 \frac{\partial Q_T}{\partial x_z} + w_1 \frac{\partial Q_T}{\partial x_z} + v_1 Q_T \frac{\partial Q_T}{\partial x_z} (Q_T + \frac{\partial Q_T}{\partial x_z})) |_{x_z=h} = 0. \]

The solutions of the stationary Schrödinger boundary problem in space confined medium or planar waveguide in the form of expansion into a series

\[ p(k, z, t) = \sum_{n}\psi_n(Z)j, \zeta_n \exp(-\lambda_n(k)\frac{t}{T}) + E_n(k, \omega) \frac{f(k, z, t)}{\lambda_n(k) + \omega} \]

Euler-Lagrange equations which linearized with respect to the polarization and displacement fluctuations acquire the following form in two-dimensional space

\[ [\alpha^* - g_3 \frac{d \psi}{dz} + g_1 k^2 \psi] \psi = E_n^2 |\psi|^2 = -\delta \tau \lambda_2 \alpha_2^2 \alpha_2^2 \]

The Euler-Lagrange dynamic equations for the order parameter component \( Q_T(k, z) \) further denoted as \( f_n(k, z) \equiv \phi_j(z, k) \) we have specified from the Lagrange function of ferroic film by varying of variables of the Functional. The boundary conditions for \( f_n(k, z) \) have the form

\[ (a f_n - g_3 \frac{\partial f_n}{\partial x_z} - w_1 \frac{\partial f_n}{\partial x_z} - v_1 f_n \frac{\partial f_n}{\partial x_z} (f_n + \frac{\partial f_n}{\partial x_z})) |_{x_z=0} = 0, \]

\[ (a f_n + g_3 \frac{\partial f_n}{\partial x_z} + w_1 \frac{\partial f_n}{\partial x_z} + v_1 f_n \frac{\partial f_n}{\partial x_z} (f_n + \frac{\partial f_n}{\partial x_z})) |_{x_z=h} = 0 \]

where \( f_n(k, z) \equiv \phi_j(z, k) = \sum_{n=1}^{m} \psi_n(k, n, v) \psi_n(Z), j = 1, 2 \) where also \( n \equiv n \) from [24] i.e. \( \psi_n(k, n, v) \psi_n(Z) = (f) \).

The solutions of Schrödinger equations can be found in the form Eq. (68) i.e. \( \psi_j(z, k) = \sum_{n=1}^{m} \psi_n(k, n, v) \psi_n(Z), j = 1, 2 \) by premultiply the equations system of Eq. (82) on \( \psi_n(Z) \) functions and by integrating the later equations system on quantum box boundaries.

Let us known the following integral relations for the found solutions algebra equation systems

\[ \int_0^1 \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = \frac{B(1-(n+k+1)+(1n+k-2n))}{(n+k)n-k)}h \]

\[ \int_0^1 \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = 0, \]

if \( n \neq k \),

\[ \int_0^1 \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = -\frac{\pi n^2}{h} \]

Since \( \int_0^1 \psi(n, Z) \frac{d}{dZ} \psi(n, Z) dZ = -\frac{\pi n^2}{h} \)

then we have replaced \( k \) in Eq. (82), keeping in the mind that the solutions of Schrödinger equations can be found in the form Eq. (68) i.e. \( \psi_j(z, k) = \sum_{n=1}^{m} \psi_n(k, n, v) \psi_n(Z), j = 1, 2 \) by premultiply the equations system of
Figure 2. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 0$.

Figure 3. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 1$. 
Figure 4. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 2$.

Figure 5. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 3$. 
Figure 6. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 4$.

Figure 7. (Color online) Normalized surface charge densities versus magnetic field applied with magnetic field strength $H_z$ in Landau gauge in nanosize ferroic SmBaMn$_2$O$_6$ with width $h = 50$nm for quantum number $n = 5$. 
I think in the article [24] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska et.al. were not allowed and consequently the quantized band energies were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96). Hence we have found the following term in equation (96) which at the magnetic field applied in Landau symmetric gauge can be transformed like after the substituting solution in equation Eq. (96) and by premultiplication on seeking distribution series of Chebyshov-Hermite functions and by integrated at defined space we have specified as

\[ g_1^2 \psi_s^n(Z) = g_1^2((k_s + \frac{\psi_s}{\psi_s})y)^2 + k_y^2, \]

where \( h = 1, v \equiv n \) does not coincide with \( n, \alpha' = \alpha' + 2\alpha'(Q_T)^2 + 3\beta'(Q_T)^3 + 4\beta'(Q_T)^4 + 7\gamma'(Q_T)^5, g_1^2 = g_1 + v_1(Q_T)^2, \) and \( (Q_T) \) is the average order parameter (for a bulk single domain sample the averaged order parameter \( \langle Q_T \rangle^2 = (\sqrt{\beta'' - 4\alpha^2} - \beta')/2\gamma' \) [24]. We have specified the depolarization field \( E_T[Z(k_s)] \) by Eq. (69) and which has been involved into Schrödinger boundary problem Eqs. (82), (83) determines the chirality form of solution of Eqs. (82), (83). Hence in addition identically true the following dispersion expressions for star of wave vector \( s_1 = 0.88*Q_s, s_2 = 0.47*Q_s, \) where \( Q_s \equiv k_s, \) the dispersion region \( M_{s_1} \rightarrow M_{s_2} \) as well as \( s_2 = 0.47/3*Q_s, \) the dispersion region \( M_{s_4} \rightarrow \Sigma_2 \) as we have specified as \( s_1 = 0.88*2/3*Q_s \) as well as \( s_2 = 0.47/2*3*Q_s, \) and the dispersion region \( \Gamma_5 \) has been found as \( s_1 = 0.88*Q_s, s_2 = 0.47*Q_s, \) where \( Q_s \equiv k_s. \)**

**Table 2. Material parameters in Landau free-energy expansion for SmBaMn_2O_6 [25].**

<table>
<thead>
<tr>
<th>Symbol and dimension</th>
<th>SmBaMn_2O_6</th>
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<td>( a_e ) (eV/A^2)</td>
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<td>( \alpha_p ) (eV/A^2)</td>
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<td>( \delta_{tb} ) (eV/A^3)</td>
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</tbody>
</table>

I think in the article [24] the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska et.al. were not allowed and consequently the quantized band energies were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96). Hence we have found the following term in equation (96) which at the magnetic field applied in Landau symmetric gauge can be transformed like after the substituting solution in equation Eq. (96) and by premultiplication on seeking distribution series of Chebyshov-Hermite functions and by integrated at defined space we have specified as

\[ g_1^2 \psi_s^n(Z) = g_1^2((k_s + \frac{\psi_s}{\psi_s})y)^2 + k_y^2, \]

the covariant derivative \( \nabla = \partial + (\epsilon \mathcal{E}_z \mathcal{A}_z) \) includes the vector potential in the symmetric gauge \( \mathcal{A}^\text{ext} = (\mathcal{A}_y, \mathcal{A}_x, \mathcal{A}_z) \) corresponding to the external magnetic field applied perpendicular to the plane along the positive z axis in which introducing \( y' = y + y_0 \) where \( y_0 = s^2 k_s, s^2 = \frac{\mathcal{A}_y}{\mathcal{E}_z} \) we have found
as well as
\[ g_s^* s^{-2} \left( \frac{-s^2}{s^2 y^2} \right) \]  
(98)

where \( \xi = s^{-1} y^* \). We seek the solution of Eq. (96) in the form
\[ \Psi_{\xi, k_1} = \left| v_{\xi, k_1} \right| \]
(100)

where \( \xi \) is a chirality, \( \psi_m(Z) = \sqrt{\frac{\pi}{h}} \sin(\pi m (\xi + \frac{1}{2})) = \sqrt{\frac{\pi}{h}} \sin(\pi m Z) \), \( Z = (\xi + \frac{1}{2}) \).

Hence in equation Eq. (96) we have found the following term
\[ g_s^* s^{-2} (\xi^2 - \frac{\xi^2}{\xi^2}) \]
(101)

after the substituting solution in equation Eq. (96) and by premultiplication on seeking distribution series of Chebyshov-Hermite functions and by integrating at defined space we have specified
\[ \int_{-\infty}^{\infty} H_m(\xi) \exp^{-\xi^2/2} \xi^2 H_n(\xi) \exp^{-\xi^2/2}, \]
(102)

Taking into the account the following recurrence relations we have specified
\[ \xi H_n(\xi) = n H_{n-1}(\xi) + \frac{1}{2} H_{n+1}(\xi), \]
(103)

\[ \xi H_{n-1}(\xi) = (n - 1) H_{n-2}(\xi) + \frac{1}{2} H_n(\xi), \]
(104)

\[ \xi H_{n+1}(\xi) = (n + 1) H_n(\xi) + \frac{1}{2} H_{n+2}(\xi), \]
(105)

\[ \xi^2 H_n(\xi) = n(n - 1) H_{n-2}(\xi) + (n + \frac{1}{2}) H_n(\xi) + \frac{1}{4} H_{n+2}(\xi). \]
(106)

Hence
\[ I = \int_{-\infty}^{\infty} \xi^2 \exp^{-\xi^2} H_n(\xi) H_m(\xi) d\xi = \int_{-\infty}^{\infty} n(n - 1) \exp^{-\xi^2} H_{n-2}(\xi) H_m(\xi) d\xi + \int_{-\infty}^{\infty} \frac{1}{4} \exp^{-\xi^2} H_{n+2}(\xi) H_m(\xi) d\xi, \]
(107)

Since for normalization integral we have found
\[ \int_{-\infty}^{\infty} \exp^{-\xi^2} H_n(\xi) H_m(\xi) d\xi = (-1)^n \int_{-\infty}^{\infty} \frac{d}{d\xi} H_n(\xi) \exp^{-\xi^2} d\xi = (-1)^{n-1} \int_{-\infty}^{\infty} H_n'(\xi) \exp^{-\xi^2} d\xi = (-1)^{n-1} \int_{-\infty}^{\infty} 2n H_{n-1}(\xi) \exp^{-\xi^2} d\xi = 2^n (n+1)! \int_{-\infty}^{\infty} \exp^{-\xi^2} d\xi = 2^n n! \sqrt{\pi}, \]
(108)

hence
\[ N = \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi) \exp^{-\xi^2} d\xi = \sum_{m=0}^{\infty} Y_{0, n,m}^{2n+1}, \]
(109)

as well as
\[ I = N(n(n - 1)\delta_{m,n-2} + (n + \frac{1}{2}) \delta_{m,n} + \frac{1}{4} \delta_{m,n+2}). \]
(110)

In addition we have specified
\[ I = \int_{-\infty}^{\infty} H_m(\xi) \exp^{-\xi^2/2} \frac{d}{d\xi} (H_n(\xi) \exp^{-\xi^2/2}) d\xi, \]
(111)

\[ \frac{d}{d\xi} (\exp^{-\xi^2/2} H_n(\xi)) = (n H_{n-1}(\xi) - \frac{1}{2} H_{n+1}(\xi)) \exp^{-\xi^2/2}, \]
(112)

\[ \frac{d}{d\xi} (\exp^{-\xi^2/2} H_n(\xi)) = ((n - 1) H_n(\xi) - \frac{1}{2} H_n(\xi)) \exp^{-\xi^2/2}, \]
(113)

\[ \frac{d}{d\xi} (\exp^{-\xi^2/2} H_{n+1}(\xi)) = ((n + 1) H_n(\xi) - \frac{1}{2} H_{n+2}(\xi)) \exp^{-\xi^2/2}, \]
(114)
\[
\frac{\partial^2}{\partial \xi^2} \left( \exp^{-\xi^2/2} H_n(\xi) \right) = n(n-1)H_{n-2}(\xi) - \frac{1}{2}H_n(\xi) \exp^{-\xi^2/2} - \frac{1}{2}((n+1)H_n(\xi) - \frac{1}{2}H_{n+2}(\xi)) \exp^{-\xi^2/2},
\]

hence

\[
I = N(n(n-1)\delta_{m,n-2} - (n + \frac{1}{2})\delta_{m,n} + \frac{1}{4}\delta_{m,n+2}).
\]

Hence in equation Eq. (96) we have found for the term \( g_1^2 k_1^2 \) the following quantization

\[
g_1^2 k_1^2 = g_1^2 s^{-2}(\xi^2 - \frac{\partial^2}{\partial \xi^2}) = 2g_1^2 s^{-2}(n + \frac{1}{2}),
\]

as well as

\[
w_1 k_1^4 = w_1 s^{-4}(\xi^4 + \frac{\partial^4}{\partial \xi^4} - \xi^2 \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \xi^2} \xi^2),
\]

\[
\xi^2 H_n(\xi) = n(n-1)H_{n-2}(\xi) + \frac{1}{4}H_{n+2}(\xi),
\]

\[
\frac{\partial^2}{\partial \xi^2} \left( \exp^{-\xi^2/2} H_n(\xi) \right) = n(n-1)H_{n-2}(\xi) - \frac{1}{2}H_n(\xi) \exp^{-\xi^2/2} - \frac{1}{2}((n+1)H_n(\xi) - \frac{1}{2}H_{n+2}(\xi)) \exp^{-\xi^2/2},
\]

\[
\xi^2 H_n(\xi) = n(n-1)H_{n-2}(\xi) + \frac{1}{4}H_{n+2}(\xi),
\]

\[
\xi^2 H_{n+2}(\xi) = (n+2)(n+1)H_n(\xi) + \frac{1}{2}H_{n+2}(\xi) + \frac{1}{4}H_{n+4}(\xi),
\]

\[
\xi^4 H_n(\xi) = n(n-1)H_{n-2}(\xi) + \frac{1}{4}H_{n+2}(\xi) + \frac{1}{4}H_{n+4}(\xi),
\]

\[
\frac{\partial^2}{\partial \xi^2} \left( \exp^{-\xi^2/2} H_n(\xi) \right) = n(n-1)\frac{\partial^2}{\partial \xi^2} H_{n-2}(\xi) - \frac{1}{2}H_n(\xi) \exp^{-\xi^2/2} - \frac{1}{2}((n+1)H_n(\xi) - \frac{1}{2}H_{n+2}(\xi)) \exp^{-\xi^2/2},
\]

\[
\xi^4 H_n(\xi) = n(n-1)\xi^2 H_{n-2}(\xi) + \frac{1}{4}H_n(\xi) + \frac{1}{2}\xi^2 H_{n+2}(\xi) + \frac{1}{4}H_{n+4}(\xi),
\]

\[
(-\xi^2 \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \xi^2} \xi^2)H_n(\xi) = (n(n-1)\xi^2 H_{n-2}(\xi) - \frac{1}{2}H_n(\xi) \exp^{-\xi^2/2} + \frac{1}{4}\xi^2 H_{n+2}(\xi) + \frac{1}{2}H_{n+4}(\xi),
\]

\[
s^{-4}(\xi^4 + \frac{\partial^4}{\partial \xi^4} - \xi^2 \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \xi^2} \xi^2)H_n(\xi) = s^{-4}(2(n + \frac{1}{2})\xi^2 - \frac{\partial^2}{\partial \xi^2} \xi^2)H_n(\xi),
\]

hence after the keeping in mind the following foregoing algebra transformations we have found

\[
w_1 k_1^4 = 4s^{-4}(n + \frac{1}{2})^2 \int_{-\infty}^{\infty} H_n(\xi) H_m(\xi) \exp^{-\xi^2} d\xi.
\]

Hence at the magnetic field applied in the symmetric gauge the equations system (96) of Schrödinger boundary problem of domain wall for data storage of thin SmBaMnO\(_2\) nanofilms can be rewritten like

\[
\Psi^{[\prime]}_{m}[n, \nu] = \left[ (a^* + 2g^2 s^{-2} (m + \frac{1}{2}) + 4s^{-4}(m + \frac{1}{2}^2 - g_2^2 \frac{\partial^2}{\partial \xi^2} I) + n(n-1)\xi^2 H_{n-2}(\xi) - \frac{1}{2}H_n(\xi) \exp^{-\xi^2/2} - \frac{1}{2}((n+1)H_n(\xi) - \frac{1}{2}H_{n+2}(\xi)) \exp^{-\xi^2/2},
\]

where \( m = 0, 1, 2, 3, ... \) are Landau quantum numbers, \( s_1 s_2 = 0.4136 \times k_1^2 = 0.4136 \times 2s^{-2}(n + \frac{1}{2}) \). In Figs. 2-7 we have presented the computation solutions of quantized normalized charges densities of the Eq. (130) stationary Schrödinger boundary problem for certain Landau quantum numbers \( m = 0, 1, 2, 3, 4, 5 \) with thickness of ferroelectric (FE) nanofilm \( h = 50nm \) of SmBaMnO\(_2\). We have found the computation solutions of order parameters of ferroelectric phase transition and suitable that FQHE in the form of quantized hysteresis loops.

In Figs. 8-10 we have presented the computation solutions of quantized normalized charges densities of the Eq. (96) stationary Schrödinger boundary problem for different thickness \( h \) of SmBaMnO\(_2\) ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition in the form of quantized hysteresis loops.
Figure 8. (Color online) Normalized surface charge densities versus transverse wave vector in nanosize ferroic PmBaMn$_2$O$_6$ with width $h = 50$ nm.

Figure 9. (Color online) Normalized surface charge densities versus transverse wave vector in nanosize ferroic PmBaMn$_2$O$_6$ with width $h = 20$ nm.
Table 3. Material parameters for S$_2$P$_2$Se$_6$ [29, 30, 34, 35, 24, 130].

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol and dimension</th>
<th>Incipient ferroelectric S$_2$P$_2$Se$_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curie-Weiss constant</td>
<td>$\alpha_T$ (×10$^6$Jm/(C$^2$K))</td>
<td>1.6</td>
</tr>
<tr>
<td>Curie temperature</td>
<td>$T_C$ (K)</td>
<td>$T_C = 193$</td>
</tr>
<tr>
<td>LGD-coefficient at $\eta^b$</td>
<td>$\beta$ (×10$^8$Jc$^{-4}$m$^5$)</td>
<td>-4.8</td>
</tr>
<tr>
<td>LGD-coefficient at $\eta^b$</td>
<td>$\gamma$ (×10$^{10}$Jc$^{-6}$m$^9$)</td>
<td>8.5</td>
</tr>
<tr>
<td>Gradient coefficient at ($\nabla \eta$)$^2$</td>
<td>$g_1$ (×10$^{-10}$C$^{-2}$m$^3$J)</td>
<td>-5.7</td>
</tr>
<tr>
<td>Kinetic coefficient</td>
<td>$w_1$ (×10$^{-27}$Jm$^5$/C$^2$)</td>
<td>1.8</td>
</tr>
<tr>
<td>Kinetic coefficient</td>
<td>$v_1$ (×10$^{-8}$Jm$^7$/C$^4$)</td>
<td>1.2</td>
</tr>
<tr>
<td>Kinetic coefficient</td>
<td>$g_3$ (×10$^{-10}$Jm$^3$/C$^2$)</td>
<td>5</td>
</tr>
<tr>
<td>Material permittivity</td>
<td>$\varepsilon_{11} = \varepsilon_{33}$</td>
<td>10</td>
</tr>
<tr>
<td>Material permittivity</td>
<td>$\varepsilon_0$</td>
<td>9.38</td>
</tr>
</tbody>
</table>

Table 4. Material parameters in Landau free-energy expansion for PmBaMn$_2$O$_6$ [25].

<table>
<thead>
<tr>
<th>Symbol and dimension</th>
<th>PmBaMn$_2$O$_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_t$ (eV/$\AA^2$)</td>
<td>-0.29</td>
</tr>
<tr>
<td>$a_b$ (eV/$\AA^2$)</td>
<td>2.77</td>
</tr>
<tr>
<td>$\alpha_p$ (eV/$\AA^2$)</td>
<td>1.28</td>
</tr>
<tr>
<td>$\beta_1$ (eV/$\AA^2$)</td>
<td>0.53</td>
</tr>
<tr>
<td>$\delta_{tu}$ (eV/$\AA^3$)</td>
<td>-0.45</td>
</tr>
<tr>
<td>$\delta_{ba}$ (eV/$\AA^3$)</td>
<td>-1.46</td>
</tr>
<tr>
<td>$\delta_{tb}$ (eV/$\AA^3$)</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Keeping in mind [19] the fixed potential at the top gate $\phi_O(x,z = -d) = V_g$, and zero potential at the bottom gate $\phi_E(x,z = h) = 0$ the continuity of the electric potential at the graphene layer $\phi_O(x,z = 0) = \phi_E(x,z = 0)$ and the equivalence of the difference of the electric displacement normal compound to the surface charge density $\varepsilon n(x)$ in graphene $D^{\nu}_0(x,z = 0) - D^{\nu}_3(x,z = 0) = e\varepsilon n(x)$. The latter condition we have specified for single ferroelectric domain wall (FDW) at contact with graphene as [19]

$$-\varepsilon_0 \varepsilon_{\delta, 0, 33} \frac{\partial \phi_{0}}{\partial z} + \varepsilon_0 \varepsilon_{\delta, 0, 33} \frac{\partial \phi_{\delta}}{\partial z} = Q_T(x, 3; 0, 0, z = 0) = e \varepsilon_{1, 2}(x),$$  \ (131)

$$-\varepsilon_0 \varepsilon_{\delta, 0, 33} \frac{\partial \phi_{0}}{\partial z} + \varepsilon_0 \varepsilon_{\delta, 0, 33} \frac{\partial \phi_{\delta}}{\partial z} \pm Q_p(x, 3; 0, 0, z = 0) = e \varepsilon_{1, 2}(x),$$  \ (132)

where $Q_T$, $Q_p$ are order parameters in Landau-free energy expansion for specify domain walls for data storage of thin PmBaMn$_2$O$_6$ nanofilms with the $P4mmm$ space group of reference data [1]. We have found that the approximate analytical expressions for the values $e \varepsilon_{1, 2}(x)$ and $e \varepsilon_{1, 2}(x)$ [19]

$$e \varepsilon_{1, 2}(T, u_m^e, k_t) \approx e_0 V_g \left( \frac{\varepsilon}{\delta} + \frac{\varepsilon T}{n_b} \right) \pm Q_T(k_t),$$  \ (133)

$$e \varepsilon_{1, 2}(T, u_m^e, k_t) \approx e_0 V_g \left( \frac{\varepsilon}{\delta} + \frac{\varepsilon T}{n_b} \right) \pm Q_p(k_t),$$  \ (134)

where $u_m$ is an elastic strain in the film, $u_m^e(h) \approx u_m^e(h)$, $h_d$ is the thickness of misfit dislocations, $d$ is the thickness of the oxide dielectric layer and $h$ is the thickness of the PE film.

The unique band spectrum of graphene leads to the unconventional integer quantum Hall effect (IQHE) [19]. It is known [19] that the $xy$-component of conductance tensor $\sigma_{xy}$ via Dirac-like spectrum of graphene and consequently additional double degeneration of a zero Landau level (LL) which is common for conduction and valence cones result in a special form of a Hall quantization $\sigma_{xy} = -\frac{e^2}{2\pi h} n v$, $v = \pm 2(2k + 1)$, where $n$ is a number of edge modes, $k = 0, 1, 2, ...$ is an integer number. The inner spin-valley symmetry in graphene leads to four-fold degeneracy of each LL but LL with $k = 0$ has an additional double degeneracy. Nonzero $k$ is given by the expression $k = \lfloor \frac{eB}{\pi n_B} - \frac{1}{2} \rfloor$, where symbol $[\ ]$ stands for the integer part of a number, $n$ is the 2D concentration of electrons in the graphene channel and $n_B = \frac{eB}{2\pi h}$ is the density of the magnetic field flux, threading the 2D surface corresponding to the degree of the k-th LL occupation. In
Figure 10. (Color online) Normalized surface charge densities versus transverse wave vector in nanosize ferroic PmBaMn$_2$O$_6$ with width $h = 2$ nm.

the article [19] have been explained theoretically the peculiarities of IQHE observed experimentally in graphene with p-n junction across the conduction channel. Hence the FDW induces a p-n junction in graphene at that the wall plane coincides with the p-n junction position [19]. The hallmark [20] of the quantum anomalous Hall (QAH) phase is the existence of chiral edge states, i.e. gapless channels at the edge propagating unidirectionally. The edge spectrum in the QAH phase with Chern number $\chi = -1$ confirms existence of one chiral channel per edge [20]. Because [20] the chirality of the edge channel is determined by the sign of Chern number the two QAH phases with $\chi = \pm 1$ must have edge channels propagating opposite directions. Consequently by tuning across the topological phase transition (via Weil Hall state (WHS)) between the two QAH phases one can switch the propagating direction of the edge channel [20]. This can be easily detected with a standard electric transport measurement [20]. Since $\mathcal{T}$ is broken in the 2D WHS state the Chern number $\chi = \int_{BZ} \Omega(q)dq$, where $\Omega(q) = -2\partial q_x u_y \partial q_y u_x$ which is the integral of Berry curvature over the BZ is typically nonzero [20]. Hence because $\Omega$ is an even function under inversion when the two Weil points are connected by $\mathcal{T}$ the two valley topological charges must be identical and we should have $\chi = 1$ or $\chi = -1$ [20].

We have investigated the transition from 2D WHS to QAH phases. Because the Weil points in 2D WHS are protected by the mirror $M_y$ breaking $M_y$ will generally remove the Weil points and open the energy gap [20]. This symmetry breaking can be easily achieved by slightly rotating magnetization vector away from the ground-state orientation [20].

6 The self-consistent stationarity quantized solutions of non-linear Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory

The polar-active properties of ferroelectric films have been calculated in Landau-Ginsburg-Devonshire theory framework [36, 34, 35, 24] with account of polarization vector gradients and surface charges and dipole layers on the interface in the conditions of stability of spontaneous polarization. Under the electric voltage feeds on the electrode or sound of scanning electron microscope the polarization redistributions have been achieved success. Under the increasing the electric voltage the local reversal of polarizations [36, 34, 35, 24] has been achieved which one lied to the creation the intergrown domains in thin films. Under domains recording the velocity of changes of extrinsic electric field $E$ was negligibly small and without loss of accuracy one can suppose $\text{rot}\cdot E \approx 0$. From Maxwell equations

$$\text{rot}\cdot E = -\frac{\partial B}{\partial t}, \quad (135)$$
Figure 11. (Color online) Quantized Polarization $P_3$ dispersion for LSMO/BFO with quantum number $n = 1, 2, 3$, and boundary parameters: $\lambda_1 = 5$ nm, $\lambda_2 = 50$ nm, $\sigma_s = 0.1$ C/cm$^2$, $P_b = 0.2$ C/m$^2$.

if we have assumed that the magnetic induction $B$ in Maxwell equation was independent quantity with respect to time then

$$\text{rot} E \approx 0.$$  

(136)

Let us known for the vector of electric field strength following expressions

$$E = -\nabla \varphi,$$

(137)

as well as for the vector of electric-flux density the following Maxwell equations

$$\text{div} \mathbf{D} = \text{div} (\mathbf{E} + 4\pi \mathbf{P}) = 4\pi \varepsilon_0 \rho(\varphi),$$

(138)

where $\rho(\varphi)$ is charges density, $\varepsilon_0$ is dielectric constant.

The ferroelectric film that occupies the region $-L < z < 0$ is transversally isotropic i.e. permittivity $\varepsilon_{11} = \varepsilon_{22}$ at zero electric field. We assume that the dependence of plane polarization components on $E_1,2$ can be linearized as $P_{1,2} \approx \varepsilon_0 (\varepsilon_{11} - 1) E_1,2$ ($\varepsilon_0$ is the universal dielectric constant) while the polarization component $P_3$ nonlinear depends on external fields. The spontaneous polarization $P_3$ is directed along the polar axis $z$. Further we assume that the dependence of polarization vector acquires the form

$$P(\mathbf{r}) = [\varepsilon_0 (\varepsilon_{11} - 1) E_1, \varepsilon_0 (\varepsilon_{11} - 1) E_2, P_3(\mathbf{E}, \mathbf{r}) + \varepsilon_0 (\varepsilon_{33}^b - 1) E_3].$$

(139)

Let us known the quasiequilibrium polarization distribution $P_3(x, y, z)$ with taking into account the gyrotropy effects which are shown to be connected with the stationary non-linear Schrödinger equation of domain wall found from Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory (in the articles [34, 35, 24] we have found the space solution of Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory only) as follows

$$i \alpha \frac{\partial P_3}{\partial t} - g(\Delta_t + \frac{\partial^2}{\partial x^2}) P_3 + \beta |P_3|^2 = -\frac{\partial \varphi}{\partial z},$$

(140)

with taking into consideration for the vector of electric field strength the following expressions $E_z = -\frac{\partial \varphi}{\partial z}$, with the following boundary conditions

$$(P_3 + \lambda_1 \frac{\partial P_3}{\partial z})|_{z=0} = -P_b,$$

(141)

$$(P_3 - \lambda_2 \frac{\partial P_3}{\partial z})|_{z=-L} = 0,$$

(142)

where we introduce $\Delta_t = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ Laplace operator. Inhomogeneity $P_b$ describes the effects of interface polarization stemming from the interface bonding effect and associated interface dipole. Hence the translation symmetry breaking inevitably in the vicinity of the any interface will give rise to inhomogeneity in the boundary conditions in Eqs. (140), (141), (142). The equations Eqs. (138), (140), (141), (142) yield the coupled system
orcid.org ρ W with depth (145) have the form ∼ where the H hence in abrupt junction approximation the space charge density near the interface of the strongly doped p-type (or ϕ p tral region of the semiconductor correspondingly. We have assumed ρ where ϕ q N P 3, ≈ e (x, y), −e e e e ϕ (x, y, −L + 0) = ϕ (x, y, −L − 0), ϕ(x, y, z → ∞) = 0, ϕ(x, y, +0) − ϕ(x, y, −0) = U b , ϵ 0 e b ϵ ε ϕ(x,y,−0) − P 3 (x, y, −0) − e e e e ϕ(x,y,0) = σ ϕ (x, y), − e e e e ϵ 0 ϵ 33 ϵ ε ϕ(x,y,−0) + P 3 (x, y, −L + 0) + e e e e ϵ 0 ϵ 33 ϵ ε ϕ(x,y,0) = σ f (x, y), where U b is the contact potential difference at the insulator-semiconductor interface, ϵ 33 is the dielectric constant of the dielectric gap between the tip and ferroelectric surface. We have assumed that polarization vector P 3 is directed along z axis that corresponds to the plane electrodes. The space charge density inside the doped p-type (or n-type) semi-infinite semiconductor has the form ρ ϕ (ϕ) = q [ p ϕ (ϕ) + N a ϕ (ϕ) − n ϕ (ϕ) − N a ] (146) is carriers charge, E F, E v, E c, E d, E t are the energies of Fermi level, valence band, conduc band, and donor and acceptor levels in the quasineutral region of the semiconductor correspondingly. We have assumed p(0) + N a (0) − n(0) − N a = 0, N a ≈ const, N a = const, and Boltzman approximation for electrons E d − E F − q φ >> k b T or holes E d − E v + q φ >> k b T lead to expressions ρ 0 ϕ ϵ ϵ 0 exp [ − momentum −1], ρ 0 ϕ ϵ ϵ 0 exp [ − momentum −1] correspondingly where p 0 and n 0 are equilibrium concentrations of holes and electrons in the quasineutral region of the semiconductor. Hence in abrupt junction approximation the space charge density near the interface of the strongly doped p-type (or n-type) semi-infinite semiconductor has the form ρ ϕ (ϕ) = q p ϕ (|ϕ| < W s ) where we assume depth W s = W sp (or ρ s = −q n 0 with depth W s = W sp). We have supposed that the ferroelectric film is wide-gap semiconductor or insulator. Hence ρ f (ϕ) ≈ 0 at region −L < z < 0. Keeping in mind the above found boundary conditions the one-dimension solutions of Eqs. (143), (144), (145) have the form

\[ \varphi(z) = -\frac{\rho_s}{2\varphi_{\Delta}}(W_s - z)^2 \varphi(W_s - z), \quad z > 0 \]  

\[ \varphi(z) = \int_{-L}^{z} -\frac{\rho_s}{2\varphi_{\Delta}}(W_s - z)^2 \varphi(W_s - z) + U_s + \frac{H}{\varphi_{\Delta}}(\sigma_s + \sigma_f - \rho_s^0 W_s), \quad -L \leq z < 0 \]  

\[ \varphi(z) = U_s + \frac{H + L + z}{\varphi_{\Delta}}(\sigma_s + \sigma_f - \rho_s^0 W_s), \quad -L - H \leq z < -L, \]  

where (146) is the Hevithaide step function.
Hence we have found the electric field \( E_3(z) = -\frac{\partial \varphi}{\partial z} \) and electrical displacement \( D_3 \) distributions based on Eqs. (146), (147), (148) in the form

\[
E_3(z) = -\frac{P_3(z)}{\epsilon_0 \epsilon_{33}} + \frac{\rho_0^W \omega^{0}_{3} - \sigma_z}{\epsilon_0 \epsilon_{33}}, \quad (149)
\]

\[
D_3(z) = \rho^0_s W_i - \sigma_z, \quad -L < z < 0, \quad (150)
\]

\[
E_3(z) = \rho^0_i (z - W_i) \theta(W_i - z), \quad z > 0, \quad (151)
\]

\[
D_3(z) = \rho^0_i (z - W_i) \theta(W_i - z), \quad z > 0, \quad (152)
\]

\[
P_3(z) = \rho^0 s \left( \frac{z}{\epsilon_0} - W_i \right) \theta(W_i - z), \quad z \geq 0. \quad (153)
\]

Hence the compensated free charge \(-\sigma_f = D_3(-L)\) has been created at second interface \( z = -L \), and the electroneutral condition has the form \(-\sigma + \rho^0_i W_i + \sigma_f = 0\). Hence we have found the self-consistent stationarity soliton solution of non-linear Schrödinger or Euler-Lagrange boundary problem for the polarization vector distributions with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory as following

\[
aP_3 + \beta P_3^3 - g(\Delta_i + \frac{\partial \varphi}{\partial z}) P_3 = -\frac{\partial \varphi}{\partial z}, \quad (154)
\]

Under variate Eq. (154) we have found

\[
a \frac{\partial P_3}{\partial z} + 3 \beta P_3^2 \frac{\partial P_3}{\partial z} - g \Delta_i \frac{\partial P_3}{\partial z} - g \frac{\partial P_3}{\partial z} P_3 = -\frac{\partial \varphi}{\partial z}, \quad (155)
\]

with the Maxwell equation in thin polar active ferroelectric film

\[
e^b = e^{b}_{33} \frac{\partial \varphi}{\partial z} + e_{11} \Delta_i \varphi = \frac{1}{\epsilon_0} \left( \frac{\partial P_3}{\partial z} - \rho_f(\varphi) \right), \quad -L < z < 0, \quad (156)
\]

after the following elementary algebraic transformations we have found

\[
-\frac{\partial \varphi}{\partial z} = \frac{\partial P_3}{\partial z} \Delta_i \varphi - \frac{1}{\epsilon_0 \epsilon_{33}} \frac{\partial P_3}{\partial z} - \rho_f(\varphi), \quad -L < z < 0, \quad (157)
\]

as well as

\[
\frac{\partial P_3}{\partial z} (\alpha + 3 \beta P_3^2) + \frac{1}{\epsilon_0 \epsilon_{33}} - g \frac{\partial P_3}{\partial z} P_3 = -\frac{\partial \varphi}{\partial z}, \quad (158)
\]

Hence under integrate transformations of Eq. (158) we have found

\[
\int dz \frac{\partial P_3}{\partial z} (\alpha + 3 \beta P_3^2) + \frac{1}{\epsilon_0 \epsilon_{33}} - g \frac{1}{\epsilon_0} \int_L^0 dz \int dz \frac{\partial P_3}{\partial z} P_3 - \int_0^L dz P_3^2 \frac{\partial P_3}{\partial z}, \quad C_1 = 0, \quad (159)
\]

under the second integrate transformation of Eq. (159) we have written

\[
P_3 (\alpha + 3 \beta P_3^2) + \frac{1}{\epsilon_0 \epsilon_{33}} - g \frac{1}{\epsilon_0} \int_L^0 dz \int dz \frac{\partial P_3}{\partial z} P_3^2 - \frac{\rho^0 W_i - \sigma_i}{\epsilon_0 \epsilon_{33}} + C_1 = 0, \quad (160)
\]

under the third integrate transformation of Eq. (160) we have derived

\[
P_3 (\alpha + 3 \beta P_3^2) + \frac{1}{\epsilon_0 \epsilon_{33}} - g \frac{1}{\epsilon_0} \int_L^0 dz P_3^2 \frac{\partial P_3}{\partial z} - \frac{\rho^0 W_i - \sigma_i}{\epsilon_0 \epsilon_{33}} + C_1 = 0, \quad (161)
\]

the equation from which one can find polarization \( P_3 \) via average polarization \( \langle P_3 \rangle \) in the form

\[
P_3 (\alpha + 3 \beta \langle P_3 \rangle^2) + \frac{1}{\epsilon_0 \epsilon_{33}} - g \langle P_3 \rangle^3 - \frac{\rho^0 W_i - \sigma_i}{\epsilon_0 \epsilon_{33}} + C_1 = 0. \quad (162)
\]

Keeping in mind the found boundary condition we have found

\[
P_3 = \frac{g \langle P_3 \rangle^3 + \rho^0 W_i - \sigma_i}{(\alpha + 3 \beta \langle P_3 \rangle^2) + 1} + C_1, \quad (163)
\]

as well as

\[
P_3 = \frac{\rho^0 s \left( \frac{z}{\epsilon_0} - W_i \right) \theta(W_i - z)}{(\epsilon_0 \epsilon_{33}) (\alpha + 3 \beta \langle P_3 \rangle^2) + 1} + C_1, \quad (164)
\]
for the coupled states of particles from the conditions that the first quantized wave function or first quantized polarization \( P_3 \) has not the node on quantum well width and keeping in mind the next boundary conditions we have found

\[
\psi = A \xi \sinh (\xi), \quad \psi_1 = A \xi \cosh (\xi), \quad \phi = A \xi \cosh (\xi), \quad \phi_1 = -A \xi \sinh (\xi),
\]

where \( \xi = \sqrt{\epsilon_0 \epsilon_3 g_\text{eff}} \), we found the two constants of integrations \( \psi(z, L) = 1 - A \xi (\xi - \xi \cosh (\xi) + \xi \sinh (\xi) \cosh (\xi/2) + \lambda \xi \cosh (\xi/2)) \).

The first energy level of an electron in the quantum well of width \( w \) is equal to [147] \( E_1 = \frac{2 \epsilon g w}{m} \), where \( m \) is an electron effective mass and \( \xi \) is determined from equation \( \cosh (\xi) = \pm \xi \), where \( \gamma = \frac{h}{w} \sqrt{2 m e^2} \tan \xi > 0 \), and \( \xi = \frac{h \xi}{2} \). For \( k_0 \) the following equality holds

\[
\arcsinh \left( \frac{h k_0}{2} \right) = \frac{n \pi - k_0 w}{w}, \quad n = 1, 2, 3, \ldots \text{ i.e is an integer number. Hence we have derived}
\]

the quantized polarization vector \( P_3 \) as well as the quantized energies.

For the average polarization \( P_3 \) we have written the following equation

\[
\langle P_3 \rangle = \rho_j W_j - \sigma_j + \frac{e_j e_j}{2 \epsilon_0} W^2 - \frac{e_j z_i}{L} (U_b + U_c + \frac{H}{\epsilon_0 \sigma_3}) \langle \sigma_j + \sigma_f - \rho_j W_s \rangle,
\]

where \( z = \frac{e_j z_i}{2 \epsilon_0} W_s \), and for the average polarization \( P_3 \) we can derive the cubic equation like

\[
\left( \alpha + \frac{1}{\epsilon_0 \sigma_3} \right) \langle P_3 \rangle + \langle P_3 \rangle^3 (3 \beta - g \langle \psi \rangle) + 3 \beta P_3 \langle \psi \rangle \langle P_3 \rangle^2 = \frac{e_j e_j}{\epsilon_0 \sigma_3} \langle \psi \rangle - P_3 \langle \psi \rangle \alpha + \frac{1}{\epsilon_0 \sigma_3} \langle P_3 \rangle^3 (3 \beta - g \langle \psi \rangle) + 3 \beta P_3 \langle \psi \rangle \langle P_3 \rangle^2 = E_b (L, H) + E_e (L, H),
\]

where \( E_b (L, H) = \frac{e_j e_j}{\epsilon_0 \sigma_3} (U_b + \frac{H}{\epsilon_0 \sigma_3} \langle \phi \rangle), \quad E_e (L, H) = \frac{e_j e_j}{\epsilon_0 \sigma_3} (U_c + \frac{H}{\epsilon_0 \sigma_3} \langle \phi \rangle), \quad E_b (L, H) = \frac{e_j e_j}{\epsilon_0 \sigma_3}, \) are built-in electrostatic field and the external field correspondingly. From the equation \( \alpha + \frac{1}{\epsilon_0 \sigma_3} \left( \frac{e_j e_j}{\epsilon_0 \sigma_3} (U_b + \frac{H}{\epsilon_0 \sigma_3} \langle \phi \rangle) \right) = 0 \)

the cubic equation for average polarization \( P_3 \) we can derive as follows

\[
\alpha + \frac{1}{\epsilon_0 \sigma_3} \left( \frac{e_j e_j}{\epsilon_0 \sigma_3} (U_b + \frac{H}{\epsilon_0 \sigma_3} \langle \phi \rangle) \right) = 0
\]

For the proper ferroelectrics the coefficient \( \alpha(T) = \alpha(T - T_C) \), where \( T = \text{absolute temperature} \), \( T_C \) is the Curie temperature renormalized by the epitaxial misfit strain \( u_m \). The strain originates from the thin film (a) and the semiconductor (c) lattice constants mismatch [111, 113, 77, 104, 94, 136, 135].

we have found Curie temperature for phase transition from ferroelectric phase into high symmetric paraelectric phase like

\[
T_{\text{phase transition}} = T_{\text{Curie}}^* = \frac{1}{\alpha_j \epsilon_3 \sigma_3} \left( 1 - \frac{e_j e_j}{\epsilon_0 \sigma_3} (U_b + \frac{H}{\epsilon_0 \sigma_3} \langle \phi \rangle) \right),
\]

where

\[
T_{\text{Curie}}^* = T_{\text{Curie}} + \frac{2 \alpha_j}{\epsilon_0} \left( 4 \epsilon_0 \epsilon_3 g_\text{eff} \right) \left( \sigma_j \right)^{-1} \left( \sigma_3 \right)^{-1} u_m \left[ 34, 35 \right].
\]

In Fig. 11 we have presented the quantized out-of-plane polarizations \( P_3 \) dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilm with semiconductor with boundary parameters: \( \lambda_1 = 5 \text{ nm}, \lambda_2 = 50 \text{ nm}, \sigma_3 = 0.1 \text{ C/cm}^2, \) \( P_3 = 0.2 \text{ C/m}^2. \)

In Fig. 12 we have presented the quantized out-of-plane polarizations \( P_3 \) dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilm with semiconductor with boundary parameters: \( \lambda_1 = 0, \lambda_2 = 0, \sigma_3 = 0 \text{ C/cm}^2, P_3 = 0.1 \text{ C/m}^2. \)

The quantized out-of-plane \( P_3 \) polarizations were clearly visible like the festive fires of candles.
### Table 5. The material parameters: Band gap, (eV), Carriers concentrations cm\(^{-3}\), Background permittivity, LGD-expansion coefficient for ferroelectrics, electrostriction, and elastic constants [34, 35, 24, 130].

<table>
<thead>
<tr>
<th>Material</th>
<th>Band gap, (eV)</th>
<th>Concentrations cm(^{-3})</th>
<th>LGD-expansion coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaSrMnO(_3) (LSMO) half-metal</td>
<td>1 p type</td>
<td>1.83×10(^{22})</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.65×10(^{21}) (La(<em>{0.7})Sr(</em>{0.3})MnO(_3))</td>
<td></td>
</tr>
<tr>
<td>BiFeO(_3) (BFO) ferroelectric</td>
<td>3</td>
<td>Wide band-gap semiconductor</td>
<td></td>
</tr>
<tr>
<td>SrTiO(_3) (STO)</td>
<td>3</td>
<td>Dielectric (without impurities)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### The self-consistent 5-polinomial quantized solutions of stationary Schrödinger or Euler-Lagrange boundary problem for the polarization vector with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory

Let us known the quasiequilibrium polarization distribution \( P_3(x, y, z) \) with taking into account the gyrotropy effects which are shown to be connected with the stationary non-linear Schrödinger equation of domain wall found from...
Table 6. The renormalization of $T_{\text{Curie}}^*$ by misfit strain and dislocations originated at critical film thickness $h_d$: $T_{\text{Curie}}^*$ at $L \leq h_d$, $T_C + \frac{h_d - 0.4}{a_2} \frac{4Q_{33}}{l_{11} + 12}$; $T_{\text{Curie}}^*$ at $L > h_d$, $T_C + \frac{h_d - 0.4}{a_2} \frac{4Q_{33}}{l_{11} + 12}$ [34, 35, 24, 130].

<table>
<thead>
<tr>
<th>Interface orientation</th>
<th>$u_m$</th>
<th>$h_d$, nm</th>
<th>$T_{\text{Curie}}^*$ at $L \leq h_d$</th>
<th>$T_{\text{Curie}}^*$ at $L &gt; h_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSMO/STO [100]</td>
<td>-0.0074</td>
<td>10.8</td>
<td>-12TCurie</td>
<td>$T_{\text{Curie}}(1 - \frac{13h_d}{L})$</td>
</tr>
<tr>
<td>LSMO/BFO [100]</td>
<td>-0.0230</td>
<td>2.4</td>
<td>0.6TCurie</td>
<td>$T_{\text{Curie}}(1 - \frac{0.4h_d}{L})$</td>
</tr>
</tbody>
</table>

Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory (in the articles [34, 35, 24] we have found the space solution of Euler-Lagrange problem in the framework of Landau-Ginsburg-Devonshire theory only) as follows

$$
t \alpha \frac{\partial P_3}{\partial t} + \beta P_3|P_3|^2 + \gamma P_3(|P_3|^2)^2 - g(\Delta_t + \frac{\partial^2}{\partial z^2})P_3 = -\frac{\partial \varphi}{\partial z}, \quad (170)
$$

with taking into consideration for the vector of electric field strength the following expressions $E_z = -\frac{\partial \varphi}{\partial z}$, with the following boundary conditions

$$(P_3 + \lambda_1 \frac{\partial P_3}{\partial z})|_{z=0} = -P_b, \quad (171)$$

$$(P_3 - \lambda_2 \frac{\partial P_3}{\partial z})|_{z=-L} = 0, \quad (172)$$

where we introduce $\Delta_t = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ Laplace operator. Inhomogeneity $P_b$ describes the effects of interface polarization stemming from the interface bonding effect and associated interface dipole. Hence the translation symmetry breaking inevitably in the vicinity of any interface will give rise to inhomogeneity in the boundary conditions in Eqs. (170), (171), (172). The equations Eqs. (138), (170), (171), (172) yield the coupled system

$$
\left(\frac{\partial^2 \varphi}{\partial z^2} + \Delta_t \varphi\right) = 0, \quad -H - L < z < -L, \quad (173)
$$

$$
e_{33}^b \frac{\partial^2 \varphi}{\partial z^2} + \epsilon_{11} \Delta_t \varphi = \frac{1}{e_0} \left(\frac{\partial P_3}{\partial z} - \rho_f(\varphi)\right), \quad -L < z < 0, \quad (174)
$$

$$
e_0 \epsilon_0 \frac{\partial^2 \varphi}{\partial z^2} + \Delta_t \varphi = -\rho_i(\varphi), \quad z > 0, \quad (175)
$$

where $e_{33}^b$ is background insulator permittivity of incipient ferroelectric, $e_i$ is the semiconductor bare lattice permittivity. The equations Eqs. (173), (174), (175) were supplemented with the boundary conditions $\varphi(x, y, -L-H) = U_s(x, y), \varphi(x, y, -L + 0) = U_s(x, y, -L - 0), \varphi(x, y, +0) - \varphi(x, y, -0) = U_b, e_{33}^b \frac{\partial \varphi(33)}{\partial z} - P_3(x, y, -L) - e_{33}^b \frac{\partial \varphi(x, y, +0)}{\partial z} = \sigma_f(x, y), -e_0 \epsilon_{33}^b \frac{\partial \varphi(x, y, -L-0)}{\partial z} + P_3(x, y, -L + 0) + e_0 \epsilon_{33}^b \frac{\partial \varphi(x, y, +0)}{\partial z} = \sigma_f(x, y),$ where $U_b$ is the contact potential difference at the insulator-semiconductor interface, $e_{33}^b$ is the dielectric constant of the dielectric gap between the tip and ferroelectric surface.

We have assumed that polarization vector $P_3^b$ is directed along $z$ axis that corresponds to the plane electrodes. Hence we have found the self-consistent stationarity quantized solution of 5-polynomial Schrödinger or Euler-Lagrange problem for the polarization vector distributions with one-dimensional Maxwell equations system at interface of thin ferroelectric films and semiconductor in framework of Landau-Ginsburg-Devonshire theory as following

$$
\alpha P_3 + \beta P_3^5 + \gamma P_3^3 - g(\Delta_t + \frac{\partial^2}{\partial z^2})P_3 = -\frac{\partial \varphi}{\partial z}, \quad (176)
$$

Under variate Eq. (176) we have found

$$
\alpha \frac{\partial P_3}{\partial t} + 3\beta P_3^3 \frac{\partial P_3}{\partial z} + 5\gamma P_3^4 \frac{\partial P_3}{\partial z} - g(\Delta_t + \frac{\partial^2}{\partial z^2})P_3 = -\frac{\partial \varphi}{\partial z}, \quad (177)
$$

with the Maxwell equation in thin polar active ferroelectric film

$$
\epsilon_{33}^b \frac{\partial^2 \varphi}{\partial z^2} + \epsilon_{11} \Delta_t \varphi = \frac{1}{e_0} \left(\frac{\partial P_3}{\partial z} - \rho_f(\varphi)\right), \quad -L < z < 0, \quad (178)
$$

after the following elementary algebraic transformations we have found

$$
-\frac{\partial^2 \varphi}{\partial z^2} = \frac{1}{e_0} \Delta_t \varphi - \frac{1}{e_0} \left(\frac{\partial P_3}{\partial z} - \rho_f(\varphi)\right), \quad -L < z < 0, \quad (179)
$$

as well as
\[
\frac{\partial}{\partial z}(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}) - g \frac{\partial^2}{\partial z^2} P_3 - \frac{\rho_\varphi(p)}{c_L^{13}} = 0.
\]

Hence under integrate transformations of Eq. (180) we have found
\[
\int dz \frac{\partial}{\partial z}(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}) = g \frac{1}{2} \int_0^L dz \int dz \frac{\partial^2}{\partial z^2} P_3 - \int_0^L dz \frac{\rho_\varphi(p)}{c_L^{13}} + C_1 = 0,
\]
under the second integrate transformation of Eq. (181) we have written
\[
P_3(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}) - g \frac{1}{2} \int_0^L dz \int dz \frac{\partial^2}{\partial z^2} P_3 - \frac{\rho_0^W - \sigma}{c_L^{13}} + C_1 = 0,
\]
under the third integrate transformation of Eq. (182) we have derived
\[
P_3(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}) - g \frac{1}{2} \int_0^L dz \int dz \frac{\partial^2}{\partial z^2} P_3 - \frac{\rho_0^W - \sigma}{c_L^{13}} + C_1 = 0,
\]
the equation from which one can find polarization \( P_3 \) via average polarization \( \langle P_3 \rangle \) in the form
\[
P_3(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}) - g \langle P_3 \rangle^3 - \frac{\rho_0^W - \sigma}{c_L^{13}} + C_1 = 0.
\]
Keeping in mind the found boundary condition we have found
\[
P_3 = \frac{g \langle P_3 \rangle^3 + \rho_0^W - \sigma}{(c_L^{13}(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}))} + C_1,
\]
as well as
\[
P_3 = \frac{g \langle P_3 \rangle^3 + \rho_0^W - \sigma}{(c_L^{13}(\alpha + 3\beta P_3^2 + 5\gamma (P_3)^4 + \frac{1}{c_L^{13}}))} + C_1,
\]
for the coupled states of particles from the conditions that the first quantized wave function or first quantized polarization \( P_3 \) has not the node on quantum well width and keeping in mind the next boundary conditions we have found
\[
\psi = A\xi \sinh(\xi), \psi_1 = A\lambda_1 \cosh(\xi), \phi = A\xi \cosh(\xi), \phi_1 = -A\lambda_2 \sinh(\xi),
\]
where \( \xi = \sqrt{\phi_1^{10} g} \) [147, 156, 144, 145, 146, 147, 177, 28, 75, 152, 151], the following trial wave functions and the following normalization condition for wave function with boundary conditions in the form
\[
\psi = A\xi \sinh(\xi) + \lambda_1 \cosh(\xi) + \xi \sinh(\xi) - \lambda_2 \sinh(\xi) = 1
\]
and polarization \( \phi = A^2 \xi (\xi \sinh(\xi) + \lambda_1 \cosh(\xi) + \xi \sinh(\xi) - \lambda_2 \sinh(\xi)) \). For the average polarization \( P_3 \) we have written the following expression
\[
\langle P_3 \rangle = \rho_0^W \sigma + \frac{c_L^{13}}{2c_L^{13}} W_s^2 - \frac{c_L^{13}}{2}(U_b + U_s + \frac{H}{c_L^{13}} (\sigma_s + \sigma_f - \rho_0^s W_s)),$$
\]
where \( z = \frac{c_L^{13}}{2} W_s \), and for the average polarization \( P_3 \) we can derive the 5-polynomial equation like
\[
(\alpha + \frac{1}{c_L^{13}})(P_3) + \langle P_3 \rangle^3(3\beta - g \langle \psi \rangle) + 3\beta P_0 \phi \langle P_3 \rangle^2 + 5\gamma P_3 \phi \langle P_3 \rangle^4 + 5\gamma \langle P_3 \rangle^5 = \frac{\rho_0^W - \sigma}{c_L^{13}} (\psi) - P_0 (\phi) (\alpha + \frac{1}{c_L^{13}}),
\]
where for average magnitudes one can find the next expressions \( \langle \psi \rangle = 1 - \frac{c_L^{13}}{\xi (\xi + \lambda_2)}, \phi = \frac{c_L^{13}}{\xi (\xi + \lambda_2)} \).
8 Conclusion

The computation solutions of order parameters of ferroelectric phase transition and applicable that Fractional Quantum Hall Effect (FQHE) in the form of nanosize quantized as well as Landau quantized hysteresis loops that were clearly visible like Fairy-tale Phoenix \([1, 2, 3]\). The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for \(Q_T, Q_P, Q_b\) with sinusoidal depolarization waves for the thin \(\text{PmBaMn}_2\text{O}_6\) as well as \(\text{SmBaMn}_2\text{O}_6\) in magnetic field applied in symmetric gauge ferroic nanofilms are specified. We have presented the computation solutions of quantized normalized charges densities of the Eq. (130) stationary Schrödinger boundary problem for different thickness \(h\) of \(\text{PmBaMn}_2\text{O}_6\) as well as \(\text{SmBaMn}_2\text{O}_6\) in magnetic field applied in symmetric gauge ferroic (FE) nanofilms. The thin \(\text{Os}_x\text{Ra}_{1-x}\text{FeO}_3\) nanofilms have quasiparticle excitations that can be described by (2+1)-dimensional Dirac theory from developed a four-sublattice model (FSM) for the analytical description of A-cation displacements in (anti)ferroelectric-antiferrodistortive perovskites of ABO\(_3\) type. We have found that FSM explains the coexistence of rhombohedral (R), orthorhombic (O) and spatially modulated phases observed by atomic-resolution scanning transmission electron microscopy (STEM) in Os-doped RaFeO\(_3\). Using this approach we atomically resolve the theoretical model of the sublattice asymmetry inherent to the case of the A-site Os/Ra cation sublattice in \(\text{Os}_x\text{Ra}_{1-x}\text{FeO}_3\) polymorphs. We have shown that this produces an unconventional form of the quantized Hall conductivity \(\sigma_{xy} = -(2e^2/h)(2n + 1)\) with \(n = 0, 1, 2,..\) \([1, 2, 3]\).

9 Summary

In the article we have found the analytical chiral solutions of stationary Schrödinger boundary problem for domain wall for data storage of nanosize \(\text{PmBaMn}_2\text{O}_6\) ferroics with incommensurately modulated sinusoidal polarization waves \(Q_T, Q_P, Q_b\). The computer solution of quantized energies or wave vectors of stationary inhomogeneous Schrödinger boundary problem for \(Q_T, Q_P, Q_b\) with sinusoidal depolarization waves for the thin \(\text{PmBaMn}_2\text{O}_6\) nanofilm are specified. We have specified the analytical quantized solutions for the band energy dispersion in nanosize ferroics in framework of Landau-Ginsburg-Devonshire (LGD) theory as well as the numerical computer calculation results for the thin \(\text{PmBaMn}_2\text{O}_6\) nanofilm have been presented. We have presented the computation solutions of quantized normalized charges densities of the Eq. (96) stationary Schrödinger boundary problem for different thickness \(h\) of \(\text{PmBaMn}_2\text{O}_6\) ferroelectric (FE) nanofilms. We have found the computation solutions of order parameters of ferroelectric phase transition in the form of quantized hysteresis loops. We have found the quantized out-of-plane polarizations \(P_3\) dispersion for LSMO/BFO interface of polar-active ferroelectric nanofilm with semiconductor with different boundary parameters. The quantized out-of-plane \(P_3\) polarizations were clearly visible like the festive fires of candles. I think in the article \([24]\) the derived of Phase diagram modeling and domain splitting in thin ferroelectric films with incommensurate phase by A.N. Morozovska et.al. were not allowed and consequently quantized energies or wave vectors were not found by means of the uncertain inference of just these similar symmetrical expressions into (68), (96).

References


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